Disentangling entanglement

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Summary sentence

Violation of the expectation values of Bell’s theorem have been taken to exclude all ‘local realistic’ explanations of quantum entanglement. A simple computer simulation demonstrates that Bell’s inequalities are violated by a local realistic model that depends only on a statistical implementation of the law of Malus in the measurement context.

Abstract.

Bell’s theorem presented two models for explaining entanglement, one based on a quantum mechanical treatment, and the other on a simple ‘local realistic’ model. Extensive experimental data have shown that the expectations of the quantum mechanical, but not those of the ‘local realistic’, model are fulfilled. On this basis, the physics community has rejected all local, realistic treatments. The success of the quantum mechanical model was based in a realistic implementation of conservation laws, allowing prediction from the transition generating them of the distribution of orientations for orthogonally correlated photon pairs. This vectorial representation then evolved to a measurement context in which those orientations were determined using polarization analyzers. Bell’s ‘local realistic’ model was unrealistic because, although it represented the same state, it did not represent the vectorial distribution, and so failed to predict the outcome. However, a simple computer model demonstrates that the experimental outcome is dependent only on the orthogonal correlation established in the transition, not on an evolution to the measurement context in which superposition justifies ‘entanglement’. As long as the law of Malus is correctly implemented at the polarizers, it extracts the ‘quantum mechanical’ distribution from the naïve representation. The result leads to a simple picture, - a correlated pair of discrete quantum entities is generated in the transition, and this intrinsic physical state evolves naturally to a measurement context while retaining the properties established in the transition. This naïve view depends only on standard conservation laws, and is sufficient to account for the outcome of experiments to test Bell’s inequalities. This model has the advantage of eliminating the tension between quantum mechanical and relativistic interpretations.

Introduction

The "particle/wave duality" discovered through Planck's famous $E = hv$, Einstein's demonstration of the consequences for the photon and for vibrational energy levels, and the extension through de Broglie's suggestion that objects of mass in the quantum range should show detectable wavelike properties, has remained at the core of philosophical debates about quantum mechanics. Because of the difficulties pointed out by Heisenberg, the behavior of such objects cannot be measured without interactions that limit the information (*1-4*). As is well recognized, these properties introduce both epistemological and ontological problems; Northrop gives a nice summary in the introduction to Heisenberg's book (*1*). In this commentary, I discuss some of these difficulties in the context of the entanglement question. My perspective comes from attempts to bring a consideration of information transfer in the biosphere into the standard thermodynamic[1] understanding. This has uncovered some interesting properties, and has led me to propose a demarcation, in which propositions that require semantic transmission without a physical framework are outside science (*5*). This raises questions in the context of the use of 'information' in description of physical systems. In most interpretations of entanglement experiments, the orthodox approach requires properties for the wavefunction, - indeterminacy, spatial delocalization, - that lead to the suggestion that 'information' is transferred faster than light in a causally efficacious process of unknown mechanism. The difficulties that arise from this assumption have colored philosophical discussions, and seeped into popular representation so that the public face of quantum mechanics has taken on a deeply metaphysical caste that misrepresents the view of most of the physics community. In this paper, I address the questions of indeterminacy and locality which underlie these problems, and demonstrate a naïve 'local realistic' model that is fully consistent with relativistic constraints, but which fulfils the expectations of Bell's quantum mechanical model.

The philosophical background

Einstein's (*6*) argument with Bohr in the mid 1930s over the entanglement question introduce the so-called "EPR paradox", which was concerned with the apparent contradictions between quantum mechanical interpretations and the superluminal constraints of special relativity. The problem was that the quantum mechanical description required a wavefunction encompassing two or more quantum objects that extended through space to arbitrary distance. The measurement of one object, in terms of properties (location, polarization orientation or spin) that reflect the energy content of the system, seemed to determine instantaneously similar but complementary properties of an entangled remote partner. In Shimony's words, "the quantum state probabilistically controls the occurrence of actual events" (*7*). This seemed to require that energy or information was exchanged faster then light. Later developments have been taken to show that Einstein's expectations of a local realistic behavior were misplaced. The ambiguities explored in (*6*) were extended to complementary spin states by Bohm (*8, 9*), and received a seminal restatement by Bell (*10, 11*), who analyzed the expectations arising from a quantum mechanical treatment of entangled states and from a 'local realistic' treatment, and proposed inequalities that were in principle open to test (*12*). Subsequent experiments of increasing sophistication and accuracy have confirmed the results anticipated from the quantum mechanical treatment[2] (cf. (*13-15*).

The "Copenhagen interpretation" of Bohr and Heisenberg (*1, 3, 16, 17*) was formulated in the context the centrality of measurement as the ontic underpinning of hypothesis, going back at least to Boltzmann (*18*), and the evolving understanding of quantized states. The early protagonists each had their own ideas, but the orthodox view has common components. Since measurement provided the link between the classical physical world and a quantum mechanical interpretation, a complete treatment was taken as demanding a formal description of the evolution between states accessible to measurement. The initiating and final states were accessible, but the evolving wave-like regime was not, and could only be tackled via mathematical models. In the case of entangled entities, the vectorial treatment led to indeterminacy of states in a common wavefunction that "evolved" in the intervening space as the particles separated. The wavefunction was claimed to provide a "complete description", but it was unclear what was meant by this. The 'quantum' part of quantum mechanics first comes into play in the quantized behavior of transitions between energy levels. The Schrödinger equation for the hydrogen atom was developed in the context of a time-independent treatment of electron energy levels, made realistic because the electronic state was constrained to a standing-wave, and conservation constraints limited the scope. The treatment was extended to include spin states, requiring conservation of angular momentum arising from the Pauli principle. The wavefunction contains classical energy terms in the Hamiltonian, but Heisenberg uncertainty considerations necessitate a probabilistic treatment, so that the ψ-function is interpreted (through $|\psi|^2$) as representing spatial probabilities (*19*). In the extrapolation to time-dependent systems, the quantized states are 'borrowed' from the description of the transition, conservation of momentum is handled through vectorial representation of orthogonal orientations, and the states are embedded in a Hilbert-space treatment (*20*) that deals with the temporal evolution in the wave-like domain. The wavefunction of the evolving system retains the probabilistic property, but constraints relating to conservation laws are included, and introduce a realistic element. For correlated entities, the linear algebraic treatment necessitates superposition of states, and Hilbert-space has to accommodate all possible configurations compatible with the constraints. As a consequence, measurement appears to serve the function of a selection, from among the more or less infinite possibilities allowed in Hilbert space, of just the few states permitted by conservation laws, in the so called "collapse of the wavefunction". Measurement of one therefore appears to determine the state of the other, seemingly pulling two entities from a delocalized condition, in which their 'entangled' properties are 'spread' continuously over the intervening space, to the discrete locality required for interaction at the atomic level.

Although Bohr is usually represented as championing the view that quantum theory provided a complete description, what he advocated was more subtle (*3, 17*):

> "The entire formalism is to be considered as a tool for deriving predictions, of definite or statistical character, as regards information obtainable under experimental conditions described in classical terms and specified by means of parameters entering into the algebraic or differential equations of which the matrices or the wave-functions, respectively, are solutions. These symbols themselves, as is indicated already by the use of imaginary numbers, are not susceptible to pictorial interpretation; and even derived real functions like densities and currents are only to be regarded as expressing the probabilities for the occurrence of individual events observable under well-defined experimental conditions."

As summarized by Jeffrey Bub "…the import of the state then lies in the probabilities that can be inferred (in terms of the theory) for the outcomes of possible future observations on the system" (*21*). For Heisenberg, the uncertainties of quantum state were potentialities, but the wavefunction was causal (*1*), and he seems to have been the main proponent of this view (*3*).

An alternative approach was that of David Bohm (*22, 23*) (see Goldstein (*24*) for a recent review). Bohmian dynamics defines the evolution of the physical configuration of the quantum entity in terms of two functions, a Schrödinger equation, with a Hamiltonian containing appropriate energy terms that account for all interactions, and a first-order evolution equation, - the so-called Guiding Equation. In this, the particle is treated as a discrete entity whose velocity is represented in terms of the quantum probability (current/density), in such a way that the probability gradient given in terms of the wave function has a "guiding" role for the evolution of the particle. The explanatory power of classical quantum dynamics is retained, but the wavefunction has a less ambiguous role, - the trajectory of a constrained quantum object appears to be directed by the probability function through a quantum potential field. This treatment has the advantage of avoiding some difficulties of the collapse of the wavefunction; - in informal terms, the quantum object has a defined position and momentum, but only goes where the wavefunction "says" it can, so that the collapse "is a pragmatic affair" (*24*). However, the wavefunction appears to have at the same time both a more causal role and a more nebulous ontological status.

The tests of Bell's theorem are taken as having resolved the discussion about the nature of entangled states in terms of non-local interpretations. The non-local picture of the quantum state is at the root of the paradoxical properties, and of the 'tension' between the quantum and relativistic views, and has engendered an extensive speculation about the philosophical and mechanistic status of the quantum world. For example, Herbert (*2*) discusses eight distinct interpretations (Wikipedia has 13), with different treatments of the nature of the underlying quantum-reality, all of which account for the experimental data satisfactorily. However, all these hypotheses fail in Popper's sense (*25*), since none provides any practical experimental test that would allow a distinction between them; - hardly a satisfactory state of affairs. Most start from the same vectorial treatment that represents of the evolving regime through a spatially dispersed common wavefunction with indeterminate states, and as a consequence they have to deal with the appearance of superluminal problems. The picture projected (*2, 4, 26*) to an outsider is of an underlying reality that lacks the substantiality of the phenomenal world of perception; the quantum theorists seem tohave lost touch with reality.

Since my own interest is from the 'information' perspective, I will start my discussion by looking at a comprehensive account by Shimony (*7*) which covers philosophical aspects. Shimony provides this view, presented as an acceptable physical interpretation (though not his favored one):

> "Yes, something is communicated superluminally when measurements are made upon systems characterized by an entangled state, but that something is information, and there is no Relativistic locality principle which constrains its velocity."

Shimony quotes Zeilinger as a champion of this view (*27*) to the effect that

> "…If we accept that the quantum state is no more than a representation of the information we have, then the spontaneous change of the state upon observation, the so-called collapse or reduction of the wave packet, is just a very natural consequence of the fact that, upon observation, our information changes and therefore we have to change our representation of the information, that is, the quantum state."

This is an odd choice of quotation, because the statement is, at face value, quite consistent with a realistic view, and could be interpreted simply as recognizing that the quantum mechanical description of the evolution of the wave-like state is a mental picture, our "representation of…the quantum state". However, many other of Zeilinger's statements (*28*) reflect a more metaphysical view that seem to better fit Shimony's interpretation:

> "…Thus, if information is the most fundamental notion in quantum physics, a very natural understanding of phenomena like quantum decoherence or quantum teleportation emerges. And quantum entanglement is then nothing else than the property of subsystems of a composed quantum systems to carry information jointly, independent of space and time…".

Fine (*29*) has suggested that the distinction between thermodynamic and informational aspects, and the notion that information transmission is not constrained by the speed of light, can be traced back to Bohr (*16*).

Information transmission and its relation to physical states

There are several aspects of this discussion I want to address, relating to the physical state, and how it can be interpreted.

Firstly, what do we mean by information? In the everyday usage, information implies a communication context, in which the message has an encoded meaning, - its semantic content. In physics, however, there seem to be several more restricted usages.

(i) In the classical realistic view as framed by Galileo, "Nature never…cares a whit whether her abstruse reasons and methods of operation are understandable to men…"(*30*). In this view, the information content of a physical system is intrinsic to the state of the system; - there is no extrinsic semantic component. We can measure properties that provide clues from which we can infer the behavior of the underlying reality. The consistency of this behavior gives us the Laws of physics.

(ii) Another usage, as in 'information content', is in terms of distinguishable states, and/or their manipulation in encoding of information. This usage is essentially thermodynamic, - either the intrinsic physical properties allow a reading of an existing non-equilibrium condition (more or less as in (i); 'information content' is synonymous with negentropy), or the states can be reordered using work to give local gradients that allow encoding of information.

(iii) Shannon's information theory (*31*) comes into the second class, but the term is always used in the context of communication, which *requires* encoding of a semantic component. Shannon was careful to note that the engineering aspects involving manipulation of physical states were distinct from the semantic components.

(iv) The usage in the quantum context is more ambiguous. For example, as noted above, the first quote from Zeilinger could be interpreted as recognizing that the outcome of entanglement experiments is simply ascribable to the intrinsic physical properties of the quantum objects, and that the resolution through measurement was of the observer's uncertainty. However, Shimony's gloss, and the second quote from Zeilinger seem to imply that an additional semantic component is imbedded in the physical state. Similar interpretations extend to higher levels of philosophical discussion. In the classical Bohr interpretation, the physical state of the entangled entities was taken to be undetermined until measured, but this term could have a range of meanings from 'unassigned' to 'indeterminate', and this has led some to suggest that reality is *dependent* on measurement (*1, 20*), or that a particular reality is *selected* by measurement (*32*). This idea has evolved in several directions. For example, the quantum character of all physical entities has been invoked in a renaissance of Plato's Forms (*33, 34*), captured in the entangled states. The permeation of information, coded in the entangled state, through space and time, has been postulated as providing an explanatory basis for many of life's mysteries, including the emergence of consciousness. This 'Platonic' interpretation has an oddly anthropocentric bent, implying that the universe performs an intentional transmitter function, with an extrinsic semantic component encoded for reception by the human species. Shimony's description, the later quote from Zeilinger, and the quantum Platonic perspective, all lean towards this anthropocentric interpretation, and lead to the view of reality at the quantum level as deeply mysterious.

The distinction between these meanings can be clarified by recognizing explicitly the difference between the usages in physics ((i) - (ii) above) and in communication (iii). Communication is involved in everyday cultural exchanges, or in evolution, and an encoded semantic component is always implicit. In these contexts, we have to consider information transmission as involving both semantic and thermodynamic components (*5*). However, information theory (*31*) pertains to the "engineering aspects" of encoding and transmission, but says nothing about the semantic content or 'meaning' of the message. This raises the question of the thermodynamic status of the semantic component. I have argued elsewhere (*5*) that the value of semantic content is not measurable in physical terms, but only becomes apparent though translational processing in a specific context. The semantic content may confer no extra thermodynamic burden, but communication is always realized in the context of encoding, transmission, and of a translational and interpretational machinery at the receiver end, each involving work terms. Although the semantic content is insubstantial, and might thereby be thought of as unconstrained by superluminal considerations, such supposition must be fanciful, because all components of information transmission, - the several physical components of the engineering side, and the semantic content of the message, - are needed if communication is to result. Whether information transmission (with a semantic component) is involved in collapse of the wavefunction, or the information read on measurement is intrinsic to the physical state, the outcome is the same. In either case, the *something* in Shimony's statement *is* constrained by 'Relativistic locality principles'. In the orthodox treatment, superluminal *communication* is excluded on grounds of statistical effects consequent on quantum randomness (*35*). In line with this, the "impossibility of superluminal information transfer" has been suggested as "one of three fundamental information-theoretic constraints from which the basic kinematic features of a quantum description of physical systems can be derived" (*36*). However, this still leaves the question of ontic status open, - what is the 'something' that confers the causality needed to justify the orthodox interpretation?

From the above discussion, we could reach the following conclusion. The descriptions of the evolving wave-like state are necessarily framed in probabilistic terms; - when Bohr said that such states are "…not susceptible to pictorial interpretation", he meant that they cannot, in the orthodox treatment, be represented by explicit models depicting discrete physical species. The quantum objects are correlated after the transition, but are said to be 'space-like entangled' because the *treatment* requires it. The non-local picture is accepted because local realistic alternatives have been ruled out by the experimental tests of Bell's theorem. Entanglement then seems to require collapse of the wavefunction, with its causal overtones. But, if "information" is taken to be the agent involved in this untangling, then two highly implausible conditions seem to be called for. (i) Semantic transmission at superluminal rates, without a thermodynamic vehicle; and (ii) (since the information transmission has been stripped of its thermodynamic support) an exchange of energy to initiate the causal process without any thermodynamic input. Neither condition makes sense within our present understanding.

Dissecting out the non-essential components

Bell (*11*) suggested two scenarios for entangled states, one quantum mechanical, and the second 'local realistic'. Entanglement experiments have shown that the expectations of the quantum mechanical approach, but not the 'local realistic' one, are accurately reflected in measurement (cf. (*11-15*)). The measurements are taken as demonstrating "the incompatibility of local realistic theories with quantum mechanics" (*7*). It seems pertinent to ask just where the quantum mechanical treatment comes into the picture, and which components are necessary for the outcome. Bell's success in prediction of the measurement outcome has cemented certain aspects of the orthodox approach into a cohesive philosophical story. The 'quantum mechanical treatment' has come to embrace a seemless whole, made up from several distinct components. These are the treatment of the transition, the representation through a common wavefunction, the Hilbert-space treatment of the evolution (*20*), the treatment of conservation constraints embedded in these features, a 'measurement context' (involving state discriminators coupled with detection devices at spatially separated stations), and a philosophical interpretation.

As an algebraically challenged outsider, I found the account of Bell's theorem provided by McHarris (*37*) more transparent than Shimony's (*7, 12*)). For his 'local realistic' model, Bell assumed the simplest form suggested by the EPR treatment (*6*), in which the states measured for the pair (spin, or photon polarization, for example) had well-defined orthogonal values (up or down, or horizontal (H) or vertical (V) polarization). The predictions of the model (in terms of what would be measured at two well-separated detectors), are expressed as expectations, giving four possible combinations of values. Since the states (Q or R, measured at one station, S or T at the other) are othogonal vectors, it is convenient to represent them numerically as +1 or –1, giving expectation values $QS + RS + RT - QT = \pm 2$. The mean values [*E(QS)*, etc.] from a statistical sample "…depend on efficiencies, errors in measurement etc., so that they are always less than (or equal to in the ideal case) the theoretical values. Putting this all together, we come up with [the 'local realistic'] version of Bell's inequality: $E(QS) + E(RS) + E(RT) + E(QT) \leq 2$" (*37*). As noted, this treatment is aimed at the predictive properties of the model, and the process of measurement is not discussed. However, all treatments seem to assume that the discriminators would partition the quantum entities according to binary expectations, - H to one bin, V to the other, for example.

The orthodox 'quantum mechanical' approach starts from the transition (*37*) - "Charlie…prepares his particles as qubits in the entangled Bell singlet state, $|\psi\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$. He sends the first qubit to Alice and the second to Bob, who proceed to make measurements … on the following observables: $Q = Z_1$, $R = X_1$, $S = (-Z_2 - X_2)/\sqrt{2}$, and $T = (Z_2 - X_2)/\sqrt{2}$. Here X and Z are the 'bit flip' and 'phase flip' quantum information matrices, corresponding to the Pauli σ_1 and σ_3 spin matrices, $X = \left(\frac{0\,1}{1\,0}\right)$, and $Z = \left(\frac{1\;\,0}{0\,\text{-}1}\right)$. It is straightforward to show that the expectation values of the pairs QS, RS and RT are all $1/\sqrt{2}$, while that of QT is $-1/\sqrt{2}$. As a result, we obtain the quantum mechanical version of …Bell's inequality: $\langle QS\rangle + \langle RS\rangle + \langle RT\rangle - \langle QT\rangle = 2\sqrt{2}$". Effectively, in the quantum mechanical approach the qubits $|01\rangle$ and $|10\rangle$ are in superposition and the treatment expresses the conservation laws in a vectorial form that lends itself to the Hilbert space representation of the evolving wavefunction. The Pauli spin matrices set up the complementary properties of the 'entangled' species, and solution determines the possible distribution of states, and the upper limit of this version of Bell's inequality ($2\sqrt{2}$ = 2.83). Consideration of "efficiencies, errors in measurement etc." in the measurement context also applies to the quantum mechanical case, so the expectation is for an outcome ≤ 2.83 after measurement.

Shimony (*7*) provides an extension of the quantum mechanical treatment that addresses the measurement context. The orientations of entangled photons are determined by using polarizing filters and application of the classical law of Malus[3]. Since this treatment is applied after the evolution, it is implicit that the physical state of the complementary pair arriving at the polarizers is that arising from the transition. The Malus' law treatment generates the Bell inequalities, but in the context of the outcome measurement. Shimony explicitly notes that Malus' law is used here statistically.

The 'local realistic' treatment fails the experimental test (*7, 13-15, 38*). The outcome correlations show values >2, a result that violates Bell's local realistic expectation. Experiments of increasing sophistication have refined the values closer and closer to the quantum mechanical expectation of <2.83. On this basis, the physics community has written off 'local realistic' explanations of entanglement; as Shimony puts it - "the *prima facie* nonlocality of Quantum Mechanics will remain a permanent part of our physical world view, in spite of its apparent tension with Relativistic locality".

A naïve interpretation

Why did Bell's 'local realistic' model fail? It is clear from the above discussion that the failure could be accounted for simply by the fact that it could not implement the conservation laws in the vectorial form appropriate to the measurement context. The difference is in the use of the Pauli principle, and a vectorial representation of orientational possibilities in the quantum mechanical case, but not in the 'local realistic' case. In the predictive sense, the quantum mechanical treatment gives the experimental result, but the local realistic treatment never could. The vectorial description allows for calculation of realistic expectations for outcomes measured using vectorial selection, - the distribution of Heisenberg's potentialities (*1*). In this sense, the predictive success depends on the non-local, indeterminate, common wavefunction through which the properties are expressed. However, potentialities are not the same as actualities; - despite Shimony's *prima* facie claim, there is nothing in the treatment that precludes the possibility that the *physical state* of each pair is that of two complementary but discrete entities. The orthogonal correlations through which conservation of angular momentum is expressed are common to both treatments, and both apply conservation of entities.

From this discussion, perhaps a more naïve view than the conventional might be worth exploring. In any evolution of the 'entangled' state in free space, delivery of the observed correlations to the measurment context would be *natural* because the operation of conservation laws would maintain correlations inherent in the initiating transition. If the transition is recognized as local, this naïve perspective translates to the expectation that the properties of any discrete quantum object (for a photon, - frequency, orientation, phase), as determined by the transition generating it, do not change during a trajectory in free space. Although this idea is essentially tautological, it will seem absurdly over-simplified to most quantum theorists, since it is contrary to the orthodox picture of superposition. However, the principle is the same as that on which our knowledge of the evolution of the cosmos is based. If photons did not retain the properties intrinsic to the transition generating them, we could obtain no reliable information from the past. Throwing this out is tantamount to a rejection of our history of the universe. Indeed, it seems to me that, as a consequence of the uncertainty principle, no theoretical representation can make epistemological claims that extend beyond the statements above.

The naïve interpretation has features in common with the pragmatic view summarized by Mermin (*39*): "The quantum state of a system is not an objective property of that system. It merely provides an algorithm enabling one to infer from the initial set of measurements and outcomes ("state preparation") the probabilities of the results of a final set of measurements after a specified intermediate time evolution". This view corresponds to that of many working physicists, and echoes earlier perspectives such as Northrop's (*1*) distinction between deterministic aspects ('strong' causality), and the 'weak' causality inherent in the probabilistic treatment of the evolutionary phase. In this pragmatic view, the arguments about the ontological status of the wave-like regime are beside the point so long as the algebra gives results compatible with conservation laws. However, in accommodating the orthodox view like this, the 'tension' between the quantum and relativistic perspectives has been swept the under the rug.

Demonstration of a violation of Bell's inequality in a simple naïve model that is both local and realistic

In order to better understand the factors involved, experiments of the Aspect (*15*) or Zeilinger (*14*) type have been modeled in a simple computer program. The program is effectively an implementation of the naïve interpretation presented above. The model consists of a 'photon source' generating correlated pairs, 'polarization analyzers' in separate detector channels, a 'coincidence counter', and plotting routines. The photon source generates pairs of photons, with the orientation of one rotate by 90^o about the axis of propogation with respect to the other. The pair is delivered, with the assignment randomized, directly to a pair of 'two-channel analyzing polarizers'. The polarizers test the probability that a photon will be transmitted in accordance with the law of Malus. However, discrimination can be set so as to be either binary or statistical (see below). The difference between the polarizers in orientation about the z-axis can be set manually, or (by default) at random. In each cycle of the program, a selectable number (1000 in the default setting) of pairs of photons are generated, and 'sent' to a 'distant' pair of analyzers. The settings of the polarizers are then established, and the yield of photons in Ordinary and Extraordinary channels is determined. This procedure mimics an experiment in which a group of photon pairs is sent to distant analyzers, with the settings determined during the time-of-flight of the group (*27*). The measurements on the photon pairs establish a mean distribution for outcomes at each particular setting of polarizers, with the mean value scaled to a range of 0-4, scoring as positive anti-correlations between detection outcome when the two polarizer are aligned. This is equivalent to the mean of measured values (QS + RT + RS - QT), calculated from all the photon pairs measured at a particular angle difference, adjusted to fit the scale (see note[3] and Supplementary materials). Yields at each station are recorded as a 'reality' check. To help establish confidence in the routines, the progam includes gadgets to allow selection of parameters, and examination of internal results (such as photon yield, assignment of kets, polarizer angle difference) and the outcome of discriminators, which can be tracked in any cycle (see Supplementary materials). One of the gadgets (the Analog option) allows display of the Malus' law outcome expected if single photons could be fractionally partitioned between the output rays (not a realistic implementation in a quantized world, but instructive). On selection of a Run option from the menu, the 'experimental' cycle is run enough times (50 in the default) to establish a set of values for polarizer difference angles that cover the full range, and plots are generated that show the correlations, and the distribution of photons between channels. The model uses only local, realistic properties. The 'evolution' of the states between the photon source and the measurement context is direct, - no quantum mechanical interpretation is implemented. For the photon source, in the 'Binary' setting the properties generate pairs of photons one with vertical and the other with horizontal polarization, all with the same relative orientation with respect to a reference coordinate system, compatible with a set of transition vectors all oriented in a fixed direction, as might be appropriate for the output of a BBO crystal[4] (*14, 40, 41*). In the 'Statistical' setting, the orientations of a pair are orthogonal, but with the 'vertical' at random about the z-axis, as might be appropriate for the stochastic population of transition vectors in an atomic Ca cascade (*15*). In either case, the photons of the pair are correlated as required by conservation of angular momentum. For the polarization analyzers, the properties are idealized as suggested by Shimony (*7*), so that the photon arriving at each station is conserved in either the Ordinary or the Extraordinary ray. In the 'Binary' setting, the discrimination is such that any photon with a probability for transmission (given by the law of Malus intensity[3]) greater than 0.5 is directed to the Ordinary ray. In the 'Statistical' setting, Malus' law is implemented statisically, by comparing the Malus' law value to a random number between 0 and 1; if the Malus' law value is greater, the photon is sent to the Ordinary ray, so that the (statistical) fraction of photons appropriate to Malus' law appears there, and the remaining fraction in the Extraordinary ray.

The essential subroutine-codes for the program in Microsoft Visual Basic 6.0 are in the Supplemental materials, which also has a fuller description of the program, and links to an executable version and full source files.

Program outcome

In this naïve model, quantum mechanics is used only to establish the complementary properties, but everything else is 'local and realistic', - the 'photons' are discrete entities that retain the properties arising from the transition. The correlations found in the program, between the 'measurement outcome' as a function of the angular difference between the two polarizers, depend mainly on the type of discrimination selected. When the polarizers are set to implement Malus' law statistically, the correlations are in violation of Bell's 'local realistic' inequality, but in agreement with the quantum mechanical expectations, whether the photon population is binary or statistical (Fig. 1A, B, C). When the discriminators are set 'Binary', so long as the photon source is in the 'Statistical' setting, the correlations are as expected from Bell's 'local realistic' model (Fig. 1D). When the Analog option is set, the differential fractional distribution gives the $\cos^2(\theta_1 - \theta_2)$ curve expected from Malus' law, even when only a single photon pair is analyzed. Outcomes from other combinations are unremarkable. The outcomes A-C are, of course, those presented in othodox texts as *requiring* a quantum mechanical treatment (cf. (*2*)).

The results demonstrate the following:

(i) A 'local realistic' model gives an outcome in accordance with experimental observations, and with quantum mechanical expectations, as long as Malus' law is correctly implemented at the discriminators (this result is implicit in Shimony's treatment[3], though not noticed there).

(ii) If a 'local realistic' model is assumed for the state of the system, use of binary discriminators will give the result expected from Bell's 'local realistic' model.

(iii) Since 'quantum mechanics' comes into the program only in the assumption that the photon pairs are orthogonal, and that assumption is implemented as a local property of each pair, established before evolution to the 'measurement context', we can exclude the proposition that a quantum mechanical treatment of the 'evolution of the wave-package' is necessary for an outcome in accordance with a quantum mechanical expectation.

The naïve model seems sufficient to account for the experimental results, as long as Malus' law is correctly implemented. Certainly, 'local realistic' models of this sort cannot be excluded on the basis of experiments similar to this simulation. The properties of the quantum objects are quite consistent with superluminal constraints. The features that appear when the polarizers are rotated to different angles are simply a necessary consequence of the orthoganal correlation of photons in a pair established in the transition. Effectively, the Malus' law behavior recovers the natural distribution function in its allocation of outcomes. Because photons are quantized, this could only be determined statistically in the experimental situation, and is demonstrated in the model when a statistical treament of Malus' law is used. The lesson to be drawn here is that the correlations established by use of polarizing filters are intrinsic to the physical state generated in the transition; that physical state can be 'local' and 'realistic', and reflect conservation principles simply. Despite claims for their importance (*14, 27*), the time of selection of polarizer orientations does not play any critical role. Timing is an issue only if the states are thought of as being in the superposition required by the orthodox treatment.

Conclusions

The main conclusion from this simple simulation is unambiguous. The elimination of 'local realistic' treatments from the entanglement debate has been based on an inappropriate interpretation of the model.

(i) Although Bell's model may be 'local', it is clearly not 'realistic' in layman's terms; - it does not implement the full vectorial consequences of the conservation laws. It is therefore not surprising that the model failed in prediction of the outcome of experiments that measured the vectorial distribution.

(ii) The 'quantum mechanical' model *was* realistic in the above sense, but the distribution function represented was simply the natural distribution for orientations of complementary orthogonal states. Since Malus' Law effectively selects the same distribution[3], there is nothing mysterious about the success of the model. The pair is not in any way *super-correlated*.

(iii) The failure of the prediction from the 'local realistic' model was compounded by the assumption that the discriminators would have reflected the binary outcome. Consequently, the experimental outcome had to be attributed to the quantum mechanical nature of the incoming state. However, the Malus' law behavior is sufficent to pull the 'quantum mechanical' distribution of states out of a 'local, realistic' population of 'photons'. Binary discrimination simply miscounts the distribution of the vectorial states measured by polarizing filters, and so generates the expectations of the 'local realistic' model.

(iv) Rejection of the 'local realistic' model involved a somewhat circular argument. The space-like extended and indeterminate nature of the entangled state was an *interpretation* based on the mathematical treatment (*20*), but it led to a picture of the common wave function as causally efficacious (*1*). The success of a non-local treatment (and failure of the 'local' one) in predicting the outcome was taken to justify the the superposition of states described in this treatment, and hence the entanglement.

An epistemological perspective

Discussions of quantum mechanics are generally introduced with a statement to the effect that the theory is perhaps the most successful achievement of physics of the past century. I have no argument with this statement. In terms of the spectacular predictive success in a wide range of applications, the orthodox treatment provides a "complete description" of our knowledge of possible outcomes in probabilistic terms (*1, 2, 17, 20*). Nevertheless, it is instructive to explore the wider implications of the conclusions above. The need for a description of the evolving states as 'entangled' comes from a preconception that the quantum entities are in superposition, and so necessarily indeterminate. The issue of indeterminacy reflects epistemic concerns about the role of measurement in pinning down the ontic status of the wave-like regime. Unfortunately, the linear algebraic treatment, framed to deal with the challenges of uncertainty, generates the superposition that seems to license the indeterminacy. But the picture is philosophically inspired, not thermodynamically required, - the siren song of the Platonic perspective (*1, 26*) has invested the algebraic elegance with a seductive actuality. Northrop's distinction between 'strong' and 'weak' causalities can be reframed in terms of the different roles of the conservation contraints and the common wavefunction. The former provide the deterministic characteristics, and the latter the 'weak' causal characteristics. Those "spooky actions at a distance" come into play only if this distinction is lost, and the 'weak' causal characteristics are thought of as implying an actual causal linkage between the evolving entities. If we accept the wavefunction as probabilistic rather than thermodynamic, and the Hilbert-space evolution as an accounting exercise, constrained by conservation laws, we do not introduce any philosophical conflict. We can state, while staying on firm epistemological ground that, although we may not have any mechanistic certainty as to what happens in the wavelike regime, or which vectorial properties pertain to which entities, we can predict the outcome with great accuracy if we understand the initiating transition and apply appropriate conservation principles. No 'collapse' of the wavefunction occurs because the wavefunction is probabalistic, - it plays no causal role and does not, except in probabalistic terms, describe a physical reality. There is no need to invoke as real those features of the evolutionary phase, - superposition, space-like extended wavefunction, etc., - that are the sprites of the mathematical treatment[5,6]. The uncertainty resolved on identification of the location and orientation of a particular state of the system is just that, - a mind thing. There is no need to actualize a superposition of states, and no requirement for superluminal exchange.

Christian (*42, 43*) has recently suggested a "Disproof of Bell's Theorem" through a description of the evolution that uses Clifford algebra. This generates the Bell quantum mechanical expectations by using a 'local realistic' representation of discrete entities; though this is under debate, Clifford algebra is claimed to avoid the superposition consequent on the conventional linear algebra. Christian concludes that "Bell inequalities must be violated, with precisely the same characteristics as they are indeed violated in experiments … by any theory that correctly implements the algebra of orthogonal directions in the physical space." I am not qualified to evaluate the algebraic niceties of Christian's treatment, but the sentiment of his conclusion seems inescapable, and consistent with both the pragmatic (*39*) and naïve views. So long as the *transition* is treated quantum mechanically, there is no mystery to the subsequent evolution; the physical behavior can be ascribed to simple conservation laws. It is not necessary to view the objects as entangled in any causal sense, - they could as well be, in Einstein's words, "spatially separate objects … independent of each other" (*29, 44*), simply correlated as demanded by conservation of angular momentum.

The above discussion is not to be taken as an attack on quantum mechanics; - abandonment of the artificiality introduced by the metaphysical extensions of the Copenhagen interpretation leaves the core of quantum mechanics intact. There's no practical requirement for a change in theoretical approach. If the correlations predicted using the orthodox approach in any of its variants (*2*) are also reproduced by the local algebra provided by Christian (*43*), it is because they correctly implement the conservation laws. The naïve hypothesis follows from the same conservation laws. As a consequence, predictive application of such correlations, for example in quantum computation, would be unaffected (*39*). However, interpretations that depend on superposition of indeterminate entities during the evolution as a basis for exploration of a statistical set of possible outcomes might be in trouble. Christian notes that "The aim of local realism is not to reject the achievements of quantum mechanics, but to assimilate them in constructing a better map of the world" (*45*), and the same could be said of the naïve approach proposed here. Such assimilation would include a resolution, at least in this instance, of the contradictions previously thought to exist between the quantum and relativistic outlooks. Although the treatment here relates to relatively simple systems, it should be obvious that for any set of states with correlated properties generated through a common transition, the same sort of considerations will also apply. While recognizing the utlility of the orthodox approach, the naïve hypothesis might be preferred as the more economical since it gives the same outcome; neither is is dependent on interpretation of the underlying ontology.

Other philosophical issues seem worth mentioning. More care should be exercised in use of 'information' in describing quantum phenomena. Either 'information' refers to a property intrinsic to the physical state, or 'information content' is a synonym for negentropy. For inanimate systems, 'information' does not involve any intentional semantic component. The naïve view presented here is quite plain, and contrasts with the "cloud-capped towers" of metaphysical interpretation erected over the entanglement question. The universe is constrained by 'laws' that we can attempt to understand, but we have no need to postulate any deeper intentionality, or any Platonic semantic component in its workings. Information transmission in the semantic sense is a property of evolving systems. The semantic component is unique to life, - a common feature of Darwinian evolution and cultural development (*5*). We human beings have evolved a phenotype through the genetic channel to a level that allows us to develop a culture, which evolves at the supra-phenotypical level. It is this latter mode that accounts for the emergence of consciousness, mind, civilization, etc., - not any mysterious interaction between intellect and a higher Platonic reality embodied in entangled states. Because the semantic component of cultural transmission requires a translational machinery for its evaluation, the evolution of ideas is necessarily *incremental*, - we have to build on what came before. Evolution in the supra-phenotypical mode provides, *over time*, a language, physical tools, abstract thinking, civil organization, and mental tools (mathematics, logic, philosophy, science, etc.) of increasing sophistication, through which we can explore the nature of the universe. In this enterprise, we necessarily stand on the shoulders of giants, but success in the long term is conditional on throwing out dud hypotheses, and it is in that Popperian (*25*) exercise that 'measurement' has its critical role.

The naïve view seems to provide a realistic account of entanglement, and one in which relativistic constraints could apply. However, in the extensive discussion of interference effects at all scales from the atomic to the cosmological, the properties of the quantum object undergoing interference in the orthodox view are taken to be delocalized. This is considered necessary to explain the interference seen when a single entity is involved in interaction with a macroscopic system. Unfortunately, it brings back into consideration the superluminal question. Perhaps such interference effects will not yield so readily to the same naïve approach, but preservation of the simplification introduced by resolving the 'tension' might be worth some effort. In contrast to the entanglement question, "each photon…interferes only with itself" (*46*), - it is the single entity involved that is rescued by measurement from delocalization, and the interference effects themselves follow classical constraints. The Bohmian interpretation allows for a local quantum object, but the superluminal problems are transferred to the guiding wave because the approach retains the characteristics of an extended wavefunction inherited from the orthodox interpretation. If the spatially extended, causal wavefunction is not sacrosanct, perhaps a more local, cohesive wavefunction could be found to encapsulate the periodic properties of the macroscopic object, without contravention of relativistic constraints.

The many different treatments of the temporal evolution of quantum mechanical systems (cf. (*2*)) all present epistemological and ontological paradoxes because of the role of measurement, and the uncertainty consequent on Heisenberg's considerations. The philosophical problems are especially acute in quantum mechanical treatments (and cosmological theories) that require postulation of other worlds or universes *inaccessible* to our measurement (*32, 34*). Then, the sorts of conservation constraints that have allowed quantum mechanics to claim its successes in our local system are distributed to domains where they can't be tested; effectively, they are no longer realistic. This lack of access to measurement allows for expansion of speculation by some arbitrary power law, but the proponents of such schemes are left skating on the thinnest of epistemological ice (*47*). To the extent that these excursions have their roots in discussion of entangled states, it might be possible to rein them in by asking if the entanglement has been disentangled from its philosophical underpinnings.

Footnotes

[1]The term "thermodynamic" is used here as a short-hand to include the well-established conservation effects that postdate the development of classical thermodynamics, and as a descriptor for physical states in which measurement can determine conformation to the laws.

[2]The phrase used in the physics community involves a confusing double negative: "violation of Bell's inequality". This means that experimental results violate the expectation (values ≤2) of Bell's 'local realistic' model, as discussed later in the text.

[3] The law of Malus intensity is given by $I = I_0 \cos^2 \theta$, where I_0 and I are incident and transmitted intensities, and θ is the difference in orientation between the electical vector of the incident light and the polarization vector of the filter. Although derived empirically, it must obviously apply statistically in any quantized context. Dirac's convention of a ket, $|\psi\rangle$, is shorthand for a treatment using complex algebra to represent orientational possibilities (see (*4, 48*) for explanation). In the orthodox treatment, as summarized by McHarris, the $1/\sqrt{2}$ term appearing in the (+1,+1,+1,–1)$1/\sqrt{2}$ values for the four outcome probabilities, is the standard normalization constant for a two-state quantum system. However, the same equation can be derived from the quantized angular momentum, $S = \hbar\sqrt{s(s+1)}$, with the spin, s, for the photon as 1. The unitless total angular momentum for two photons is then $2\sqrt{2}$; the Bell mean quantum mechanical expectation is seen as an expression of conservation of angular momentum. The quantum mechanical distribution can be obtained from the 'local realistic' expectations simply by distributing the total angular momentum evenly among the four states represented by the 'local realistic' values for pairs QS, RS, RT, QT (to give the same vectorial values as the quantum mechanical model, (+1,+1,+1,–1) $1/\sqrt{2}$). Shimony's treatment is based on the expectation of outcomes for orthoganal pairs arriving at the two polarization analyzers, set with differences in angle, φ, expressed via Malus' law. If the trigonometric terms (+1,+1,+1,–1)

($cos^2\varphi$ - $sin^2\varphi$ = $cos2\varphi$ = $cos(\pi/4)$ = 0.7071), arising from choice of values for φ in his treatment, are replaced by their equivalent unit circle value, ($1/\sqrt{2}$ = 0.7071), we get the same equation. All treatments give the same expectation values because they all deal with the same physical state, and either implicitly or explicitly, with the same set of conservation constraints. In the orthodox treatment, solution of the Pauli matrices deals with conservation of angular momentum, and the normalization constant deals with conservation of entities. In Bell's 'local realistic' inequality, conservation of angular momentum is implicit in the orthogonal assignment, and conservation of entities is handled through the enumeration of states. The trivial transposition above then makes the former explicit, and demonstrates the match to the quantum mechanical expectation. In the Shimony treatment, the same values are derived from the physical state arriving at the measurement context. He assumes this to be the state described by the orthodox treatment, but the *in silico* simulation demonstrates that the local realistic representation is equivalent.

[4]The output of the β-barium borate (β-BaB_2O_4, BBO) crystal (*40, 41*) is determined by conservation of both angular momentum and energy: "The so-called phase-matching conditions of quantum optics…imply that the momenta and the energies of the two created photons have to sum up to the corresponding value of the original pump photon inside the crystal…The two emerging photons are entangled both in energy and in momentum" (*27*). If a single energy at half the frequency of the incident light is selected by a narrow-band interference filter, the output is two cones of light of the same angle, diverging symmetrically about the axis of incidence, labeled the ordinary and extraordinary. In the Kwiat-Zeilinger experiments (*27, 41*), the entangled pairs were sampled at intersections of the two cones, and the orientational distribution of the photon population was 'tuned' using differential refractive elements to give the four EPR-Bell states (*49*):

$$\begin{aligned} |\psi^{\pm}\rangle &= (|H_1,V_2\rangle \pm |V_1,H_2\rangle)/\sqrt{2}, \\ |\phi^{\pm}\rangle &= (|H_1,H_2\rangle \pm |V_1,V_2\rangle)/\sqrt{2} \end{aligned}.$$

[5] In the quantum mechanical interpretation, each entangled pair has a particular set of potentialities, - arising from superposition and an indeterminate assignment of attributes, - until the moment of measurement. On measurement, the assignment is determined so that measurement of the property of one state instantaneously establishes that of the other. In the Ca cascade, the two emmitted photons are of different colors (*15*). In experiments using UV-pumped BBO crystals (*27, 41*), the photon pairs could also be prepared with different colors (see note 4). In both types of experiment, the property used to test correlations has been polarization, but either attribute could in principle be measured to determine the quantum state. In both types of experiment, the H/V correlations were measured using polarizers. However, in the absence of selection, different colors occupy cones of different angle, and 'entangled' pairs could be selected from the intercepts of two cones, of complementary colors, one in the ordinary and the other in the extraordinary population. The 'potentialities' could therefore have been discriminated by dichroic filters on the basis of color, to give a binary selection for photons well-separated in energy. If the superposition model pertained, both types of measurement should 'collapse the wavefunction' and give a 'quantum mechanical' outcome. If the correlations were as in the naïve model, the H/V correlations would give a quantum mechanical outcome, but the color measurments a local realistic one.

The properties of the BBO crystal beg the question of whether further experiments are really needed. It is obvious from their trajectories in cones of different angle that (*contra* indeterminacy) the energies of the photons in ‘entangled’ pairs are determined in the transition, and established before evolution to the measurement context. From the partitioning into ordinary and extraordinary cones, and the use of differential refraction in ‘tuning’ of the photon pair (*41*), the polarization state is also determined before evolution. The ‘uncertainty’ involved is that arising from the mixing of two well-defined discrete populations, and the success of the orthodox treatment in ‘untangling’ the mixture is testimony to the arguments of the paper.

[6] Mermin (*50*) quotes a remark by Adler (*51*) to the affect that “Most theoretical physicists are guilty of…failing to distinguish between a measurable indeterminacy and the epistemic indeterminability of what is in reality determinate.”

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Figure Legends

Figure 1. The 'experiment' is set up as described in the text. The coincidence count as a function of the difference between the polarizer angles is shown in the left-hand panel, on a scale 0-4, with anti-correlation for hits in the ordinary or extraordinary ray counted positive when the two polarizers are aligned. The angle difference is normalized to absolute value, and the range 0^o to 90^o, to avoid redundancy. The solid curve shows $\cos^2\Delta\theta$, where $\Delta\theta$ is the difference in angle between the two polarizers, - the result expected from Bell's 'quantum mechanical' model (*2, 7*). The diagonal straight line is the result expected from Bell's 'local realistic' model.

The right-hand panel shows the mean yield of hits in each detector (blue circles for detector 1, yellow for detector 2), and their mean (black). When the Analog option is selected (as in **C**) the panel shows the fractional distribution expected from the law of Malus value (see Supplementary material for explanation).

The images A-D are program windows showing results with different settings, as follows:

A. Photon source is set 'Binary', Malus' law is implemented correctly ('Statistical' mode), polarizer 1 is fixed ('Fixed' mode). The emulation is of a Zeilinger-type experiment. The outcome correlation follows the curve expected from Bell's quantum mechanical expectations.

B. Photon source is set 'Statistical', Malus' law is implemented in 'Statistical' mode, polarizer angles are both randomly set (the result with polarizers in 'Fixed' mode would look similar). The correlation follows the curve expected from Bell's quantum mechanical expectations, with more noise than in **A** because of the additional statistical uncertainty.

C. Photon source is set 'Binary', discriminators were set 'Statistical', then 'Analog' was selected, Polarizer orientation set 'Manual', and polarizer 2 rotated 7.5^o between successive runs. The excellent noise level reflects an averaging of 20 experiments with 1000 'photons', but acceptable data can be generated with smaller numbers for average. The statistical implementation (left panel) of the law of Malus reveals the curve expected from Bell's quantum mechanical model. The analog implementation (right panel) shows no 'noise', indicating that the same unique value for fractional distribution was generated from each photon at each angle difference. The same curve would have been generated if a single photon had been analyzed at each setting.

D. Photon source is set 'Statistical', discriminators are set in 'Binary' mode, polarizer angles are both randomly set ('Random' mode). The result with polarizers in 'Fixed' mode would look similar. The points for correlation values fall on the diagonal, as expected from Bell's 'local realistic' model.

Figures

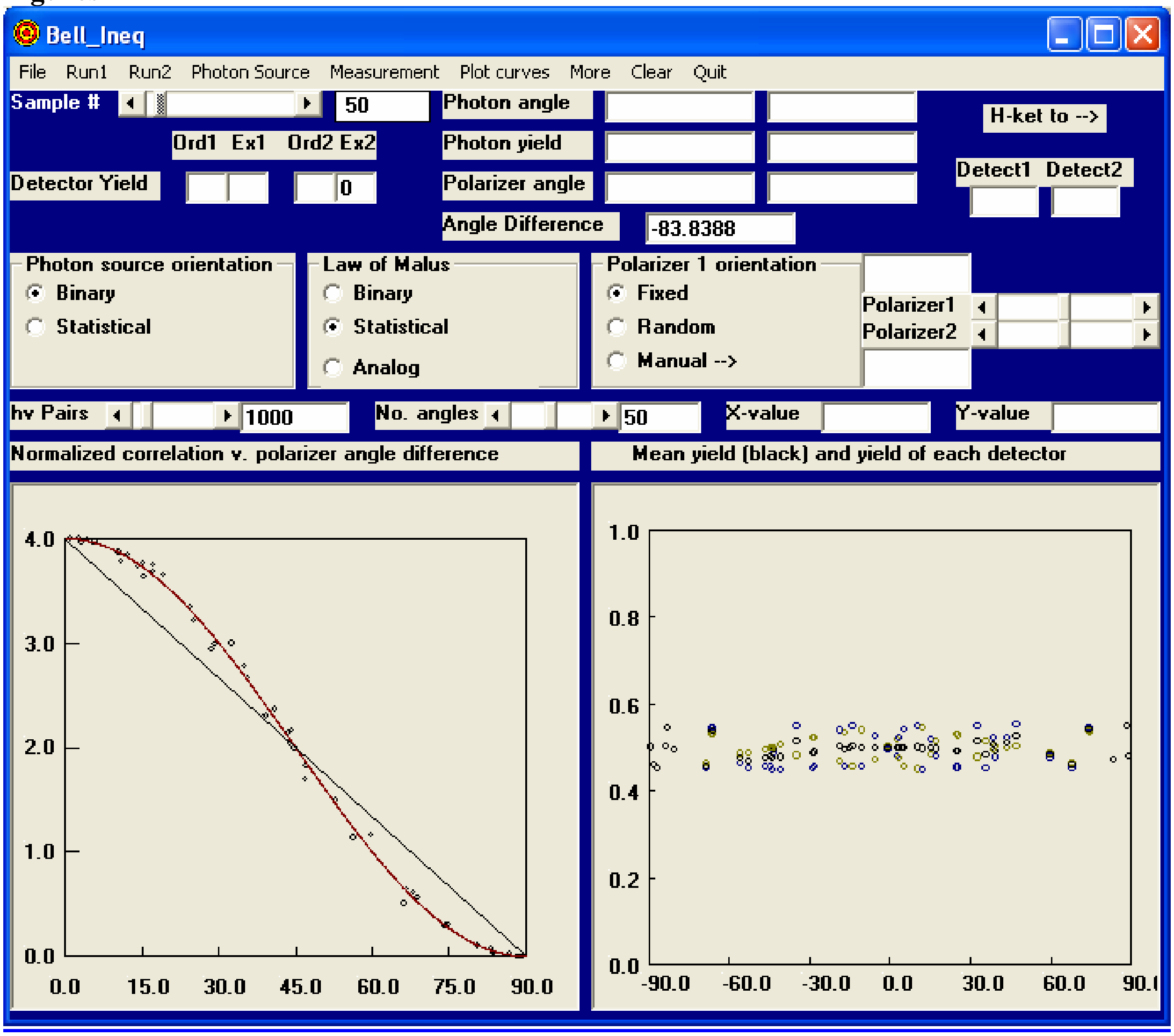

Figure 1 A

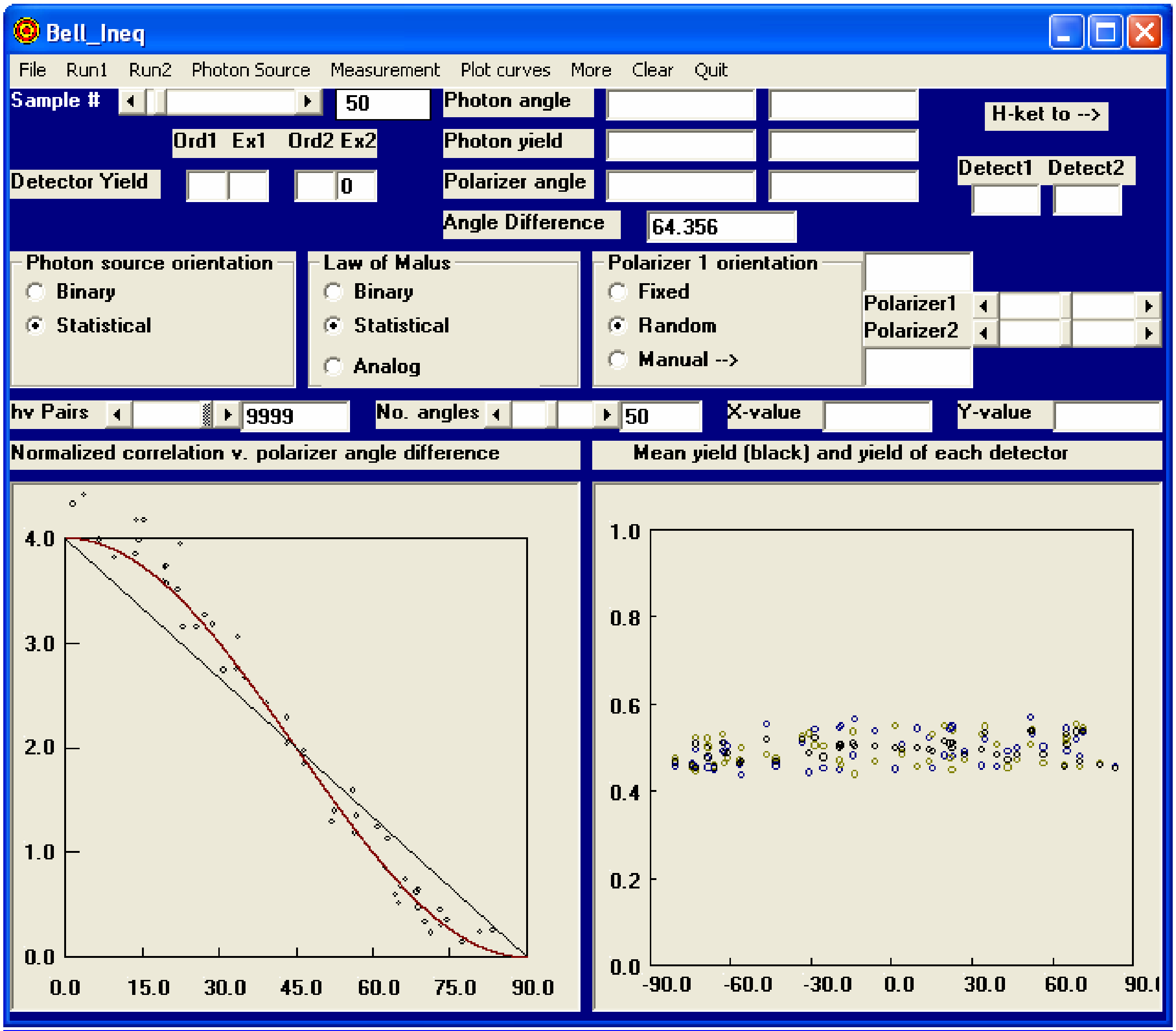
Bell_Ineq
File Run1 Run2 Photon Source Measurement Plot curves More Clear Quit
Sample #
50
Photon angle
H-ket to -->
Ord1 Ex1 Ord2 Ex2
Photon yield
Detector Yield
0
Polarizer angle
Detect1 Detect2
Angle Difference
64.356
Photon source orientation
Binary
Statistical
Law of Malus
Binary
Statistical
Analog
Polarizer 1 orientation
Fixed
Random
Manual -->
Polarizer1
Polarizer2
hv Pairs
9999
No. angles
50
X-value
Y-value
Normalized correlation v. polarizer angle difference
Mean yield (black) and yield of each detector
4.0
3.0
2.0
1.0
0.0
0.0 15.0 30.0 45.0 60.0 75.0 90.0
1.0
0.8
0.6
0.4
0.2
0.0
-90.0 -60.0 -30.0 0.0 30.0 60.0

Figure 1 B

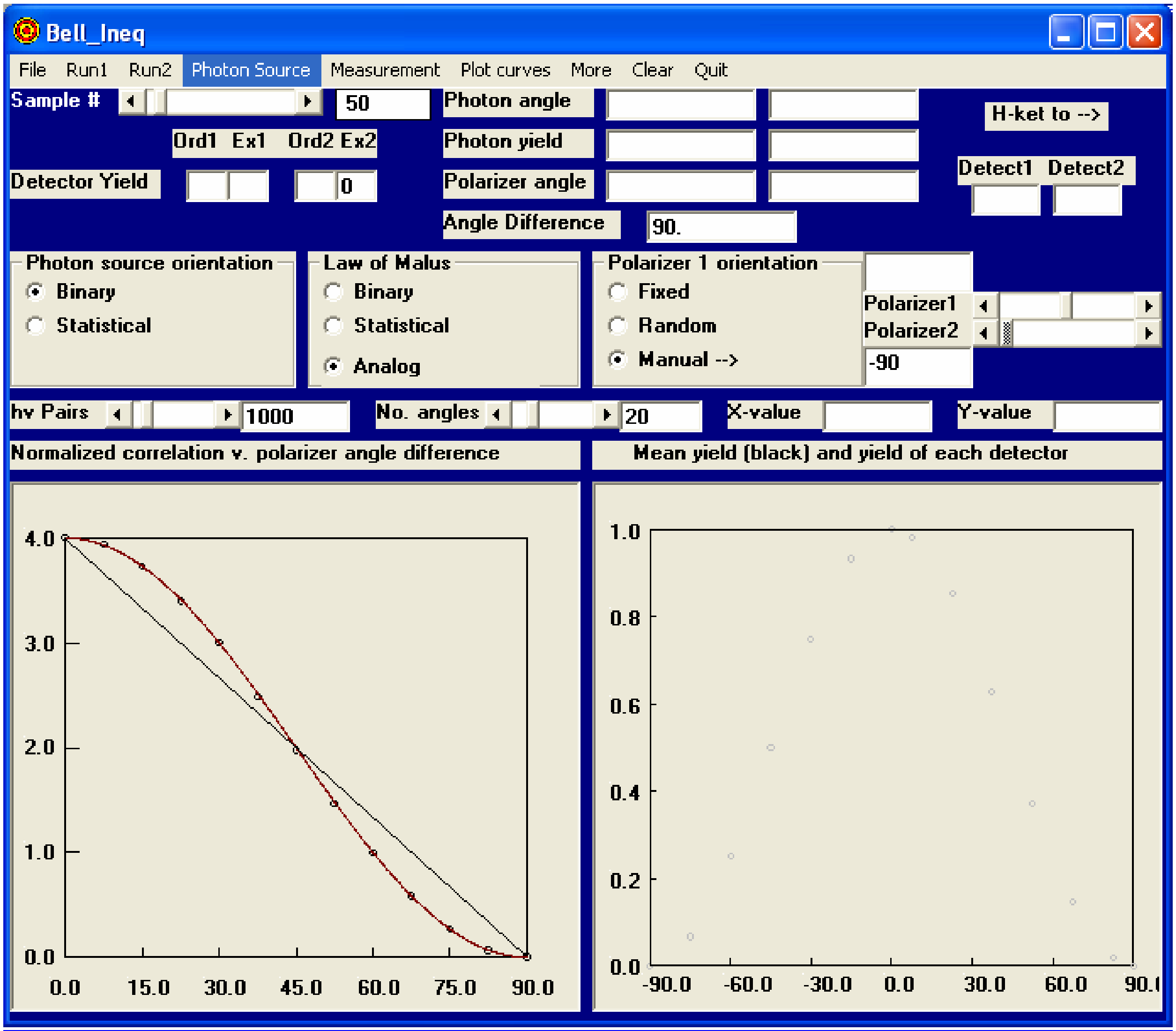
Bell_Ineq
File Run1 Run2 Photon Source Measurement Plot curves More Clear Quit
Sample #
50
Photon angle
H-ket to -->
Ord1 Ex1 Ord2 Ex2
Photon yield
Detector Yield
0
Polarizer angle
Detect1 Detect2
Angle Difference
90.
Photon source orientation
Binary
Statistical
Law of Malus
Binary
Statistical
Analog
Polarizer 1 orientation
Fixed
Random
Manual -->
Polarizer1
Polarizer2
-90
hv Pairs
1000
No. angles
20
X-value
Y-value
Normalized correlation v. polarizer angle difference
Mean yield (black) and yield of each detector
4.0
3.0
2.0
1.0
0.0
0.0 15.0 30.0 45.0 60.0 75.0 90.0
1.0
0.8
0.6
0.4
0.2
0.0
-90.0 -60.0 -30.0 0.0 30.0 60.0

Figure 1C

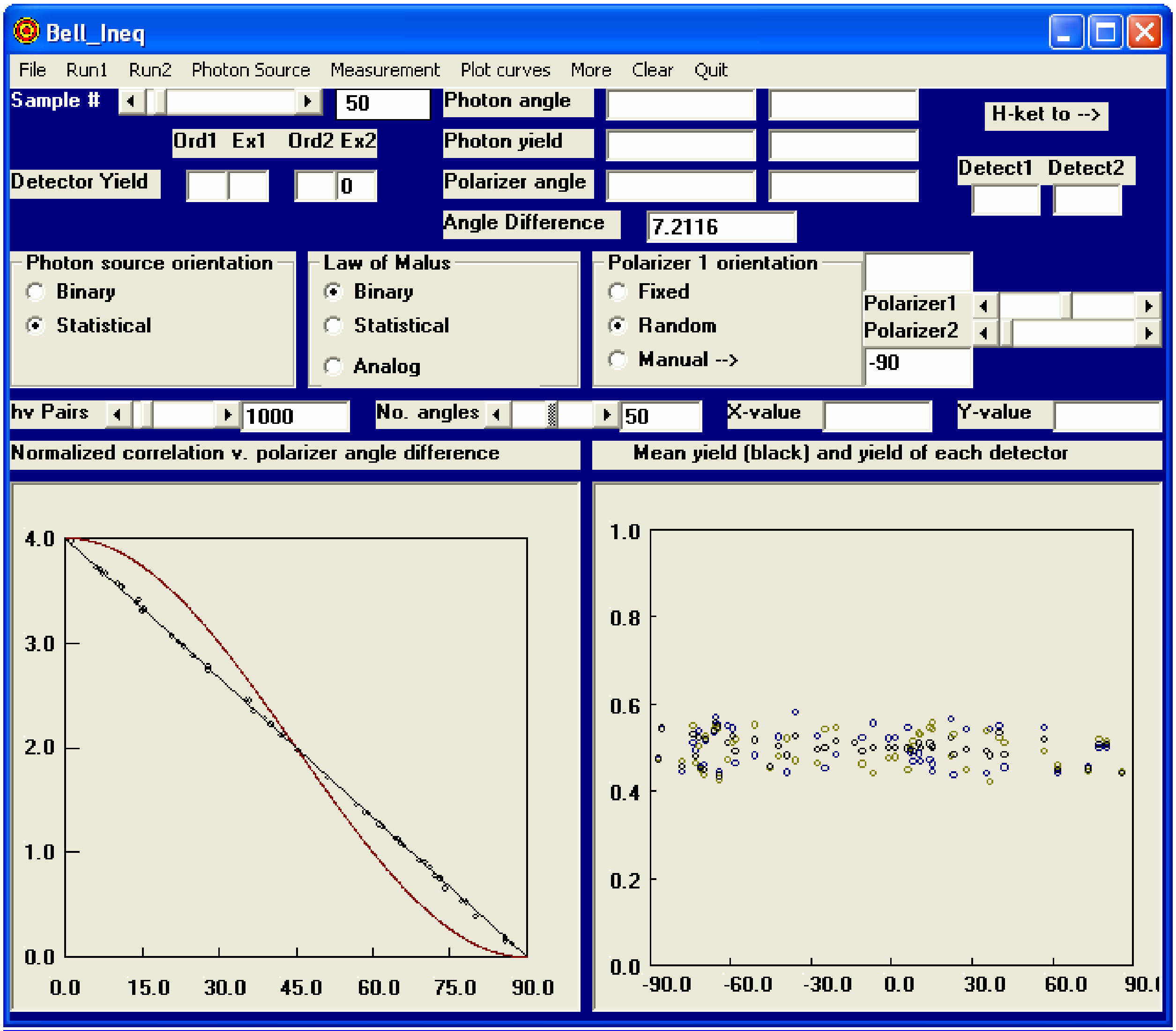
Bell_Ineq
File Run1 Run2 Photon Source Measurement Plot curves More Clear Quit
Sample #
50
Photon angle
H-ket to -->
Ord1 Ex1 Ord2 Ex2
Photon yield
Detector Yield
0
Polarizer angle
Detect1 Detect2
Angle Difference
7.2116
Photon source orientation
Binary
Statistical
Law of Malus
Binary
Statistical
Analog
Polarizer 1 orientation
Fixed
Random
Manual -->
Polarizer1
Polarizer2
-90
hv Pairs
1000
No. angles
50
X-value
Y-value
Normalized correlation v. polarizer angle difference
Mean yield (black) and yield of each detector
4.0
3.0
2.0
1.0
0.0
0.0 15.0 30.0 45.0 60.0 75.0 90.0
1.0
0.8
0.6
0.4
0.2
0.0
-90.0 -60.0 -30.0 0.0 30.0 60.0

Figure 1 D

Supplementary material

Explanation of program

The program simulates an experimental set-up to test Bell's theorem. The outline of the program in the text provides a brief description of the main functions, and perusal of the program code (below) will provide details. The functions requiring further explanation are the Menu items, Controls, and various gadgets that allow examination of internal values, and display of additional information.

Controls

Photon source controls. The 'experiment' is set up with the photon source selected as either 'Binary' or 'Statistical':

Binary mode. In 'Binary' mode, each photon of a pair is either horizontally (angle 0^o) or vertically polarized.

Statistical mode. In 'Statistical' mode, the photons of the pair are at 90^o, but the pair is rotated at random about the axis of propagation.

In either mode, photons of a pair are sent to discriminators at random.

Polarizer controls. Three different options can be selected for the polarizer orientations:

Fixed mode. When 'Fixed' is selected, angles are set so that polarizer 1is horizontal (at 0^o in the reference frame) and polarizer 2 is rotated to a random angle in the range $\pm 90^o$ for each cycle of measurement.

Random mode. When 'Random' is selected, polarizer 1 is rotated at random through the range $\pm 45^o$, and polarizer 2 is rotated at random through the range $\pm 90^o$.

Manual mode. The orientations of the two polarizers can be set using the sliders to the right, and the angle values are displayed in the associated text boxes. A 0^o angle is nominally horizontal. In Manual mode, the correlations at fixed values of polarizers are displayed as the mean value, rather than as a separate value for each cycle. This is appropriate for comparison with the standard presentation of experimental data.

Law of Malus controls. The discriminator function can be set either to implement the law of Malus ('Statistical' mode), or to operate as a binary discriminator ('Binary' mode). An additional 'Analog' option is explained below.

Statistical mode. In the 'Statistical' setting, Malus' law is implemented statisically, by comparing the Malus' law value to a random number between 0 and 1; if the Malus' law value is greater, the photon is sent to the Ordinary ray, so that when applied to a statistically significant numer of photons, the (statistical) fraction of photons appropriate to Malus' law expectations appears there, and the remaining fraction in the Extraordinary ray.

Binary mode. In the 'Binary' setting, the discrimination is such that any photon with a probability for transmission (based on Malus' law) greater than 0.5 is directed to the Ordinary ray, otherwise to the Extraordinary ray.

Analog option. The 'Analog' option in the middle group needs further explanation. When this is selected, it leaves the controls in the other groups unaffected, but freezes the other options in the Law of Malus group so that they function as set before the selection of the Analog option. However, the display in the right-hand picture is changed so that it now shows the analog version of the yield difference, ΔI, between the outputs of the two polarizers (see Fig. S1, right), given by the absolute value of the difference between the Malus' law yields (as displayed in the Photon yield text boxes). This is effectively $\Delta I = I_0\cos^2(\theta_1 - \theta_2)$. For photon pairs in Binary mode this gives a simple curve, which follows the quantum mechanical expectations. For photons at random orientations, the points are distributed at random between ± this maximal Malus' law value (with some phase shift), but normalized so that the display is in the range between 0 and the maximal value.

Displays

Left panel. The coincidence count as a function of the difference between the polarizer angles is plotted in the left-hand panel, on a scale 0-4. Since the polarization analyzers are treated as ideal, only non-redundant correlations were scored. Anti-correlation for hits in the Ordinary rays of the two polarizers (or in the two Extraordinary rays), are counted positive. This gives a count of 2 when the two polarizers are aligned and 0 when they are orthogonal. The count is multiplied by two for comparison with experimental outcomes (where none of the correlations can be treated as redundant because of non-ideal behavior) to give a range 0-4. The angle difference is normalized to absolute value to avoid redundancy. The solid curve shows $\cos^2\Delta\theta$, the result expected from Bell's 'quantum mechanical' model, where $\Delta\theta$ is the difference in angle between the two polarizers. The diagonal straight line is the result expected from Bell's 'local realistic' model. When **More** is selected from the menu, additional information is displayed (see **Gadgets).**

Right panel. The right-hand panel shows the mean yield of hits in each detector (blue circles for detector 1, yellow for detector 2), and their mean (black). When the **Analog** option is set, yield difference as a function of the difference in polarizer orientation is plotted (see above).

The two text boxes labeled **X-value** and **Y-value** display the position of the mouse pointer when the mouse is clicked on either of the displays.

Menu items

A population of photon pairs can be generated by selecting **Photon Source** from the menu. Selection of **Measurement** runs a measurement cycle at the angle difference of the current polarizer settings, using each photon pair of the population. Selection of **Plot curves** plots the single point resulting from the last 'Measurement'. The results of a single cycle can be examined using the various gadgets, as discussed below.

Selection of **Run1** from the menu initiates an 'experiment' in which a selectable number (through the **hv Pairs** slider, left of control panel) of photon pairs are generated (using the Photon source subroutine). Each photon pair is then measured (using the Measurement subroutine) using the pre-selected mode, with polarizers set at the angle difference shown in the text box (which changes as each cycle is run), and stored for plotting (using the Plot curves subroutine) before a new cycle is initiated with a different set of photon pairs, measured with a new angle difference. **Run2** initiates a similar set of 'measurements', except that the population of photon pairs is not changed before the angle difference is reset, with a small saving of time. The results are essentially the same in both modes, but Run1 is more 'realistic'. **More** allows display of the Bell expectation values (see Gadgets). **Clear** clears the two display panels. **Quit** exits the program. **File** allows the user to select or set a filename to store in text format the x- and y-values resulting from the last **Run**, as displayed in the left-hand panel.

Gadgets

The top set of gadgets (see Fig. S1) allows examination of the results of the last set of measurements. The slider labeled Sample # sets the pointer to arrays containing stored data and information that is then displayed in the different text boxes. These give:

Sample #	The slider labeled Sample # sets the array pointer to stored data and information, which is then displayed in the different text boxes. These give:
H-ket to →	The allocation of the nominally horizontal ket to either of the measurement stations (0 = not allocated, 1 = allocated).
Photon Angle	The orientation (with respect to the horizontal reference 0^o) of each photon as it arrives at the polarizers (as determined by the allocation shown in H-ket →.
Polarizer Angle	The orientation of each polarizer (with respect to the horizontal reference 0^o).
Photon Yield	The Malus' law value, determined from the difference in angle between the photon electrical vector and the polarizer orientation, as shown in the two preceding gadgets, for each of the two polarizers.
Angle difference	The difference between the angles of the two polarizers.
Detector Yield.	Shows the allocation of photons to the Ordinary (Ord1, Ord2) and Extraordinary (Ex1, Ex2) bins for the two detectors, based in the discriminator function selected, and the Malus' Law value.
More (from menu)	Choice of **More** from the menu turns on display of additional information in the left panel. The points show, at each value for polarizer angle difference, the Bell expectation values for *n* photon pairs, in different formats:

Magenta circles – Bell's local realistic mean expectations, using $\left(\sum_{i=1}^{n}\left|(\mathrm{QS}+\mathrm{RS}+\mathrm{RT}-\mathrm{QT})_{\mathrm{i}}\right|\right)/n$.

Red circles – Bell's quantum mechanical mean expectations, using $\left(\sum_{i=1}^{n}\left|(\mathrm{QS}+\mathrm{RS}+\mathrm{RT}-\mathrm{QT})_{\mathrm{i}}\times\sqrt{2}\right|\right)/n$.

Gray circles – the mean of values for the Bell local realistic outcome, using $2+\left(\sum_{i=1}^{n}(\mathrm{QS}+\mathrm{RS}+\mathrm{RT}-\mathrm{QT})_{\mathrm{i}}\right)/n$, the 2 being an offset to bring the range of values (±2) to the same scale as the other plots.

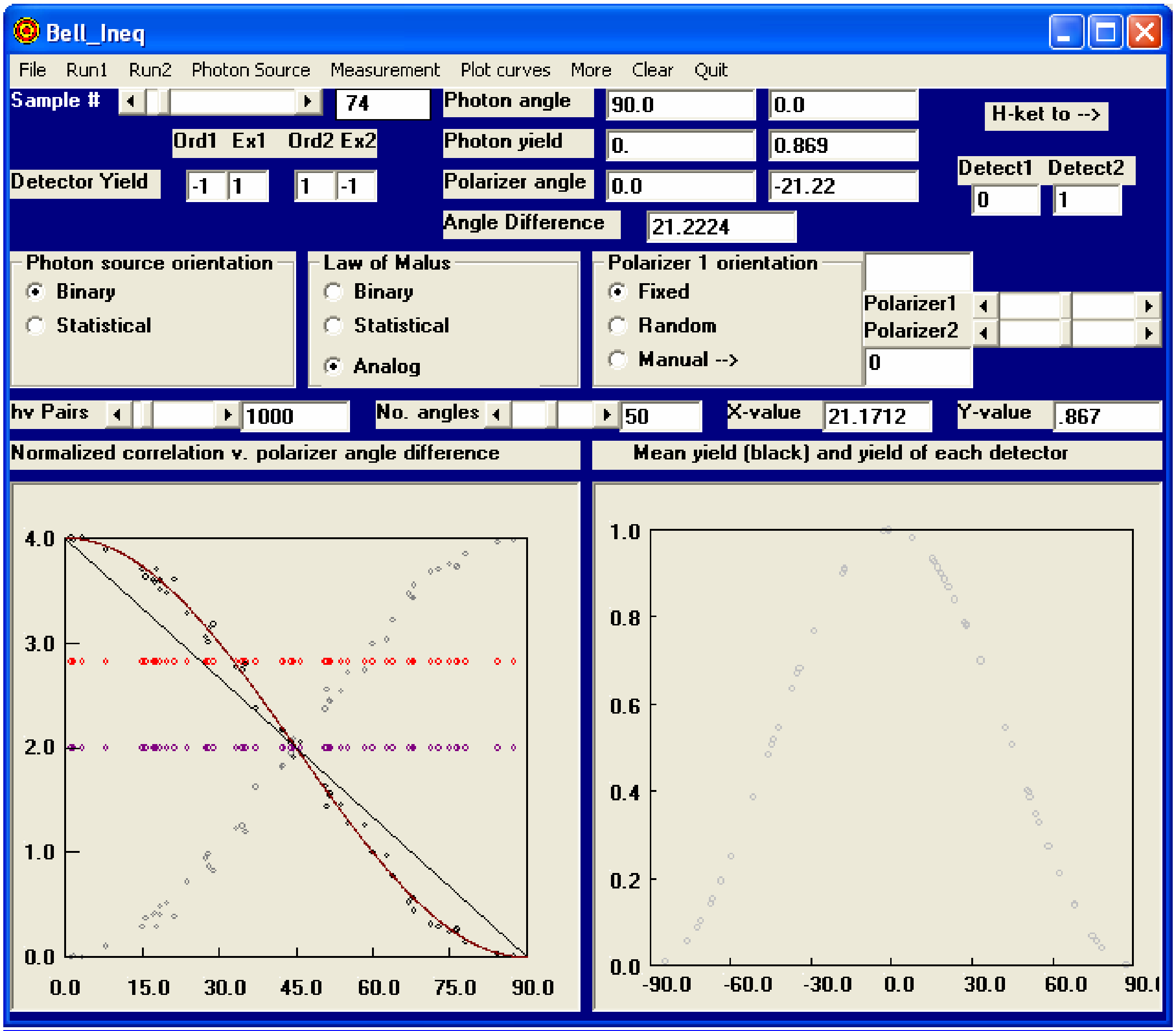

Figure S1. The program window captured after an 'experiment' to illustrate the gadgets. The 'experiment' type was **Run1**, at the settings shown. **Left panel:** The black circles show the outcome as correlations (see above), and the black lines show the curve expected from the Bell quantum mechanical expectations ($\cos^2(\Delta\theta)$ curve), and the Bell 'local realistic' expectations (diagonal). The previous setting in the Law of Malus control box was for Statistical mode, so this was the option controlling implementation (see notes above on **Analog** option). **Right panel:** The points show the absolute value of the difference between photon yields at the two polarizers given by application of the law of Malus to each photon pairs. The X- and Y-value text boxes show values returned by clicking on the panel close to the point corresponding to the angle difference shown. The **More** option had been selected from the **Menu** so the Bell expectations are also displayed in the **left Panel** (red circles, - Bell QM mean; magenta circles, Bell 'local realistic' mean; gray circles, the mean for (QS + RT + RS - QT) at each angle difference). The other gadget displays show the internal values associated with 'measurement' of the 74th photon pair of the population generated by the photon source on the last cycle of this **Run**.

Microsoft **Visual Basic V. 6 source code** (complete source files are available on line at http://www.life.uiuc.edu/crofts/Bell_Ineq/)

```
Dim ActionFlag, RunNo, Active, Correl_Flag, All_Outcomes_Flag As Integer
Dim Ket1(10000), Ket2(10000), QS, QT, RS, RT As Integer
Dim PhotonAngle1(10000), PhotonAngle2(10000)
Dim Mark(10000, 2), Cos2Theta(10000)
Dim PolarizerAngle1, PolarizerAngle2, AngleDiff
Dim PhotonYield1(10000), PhotonYield2(10000), BellCount1(10000), BellCount2(10000)
Dim Detector1Ord(10000), Detector1ExOrd(10000), Detector2Ord(10000),
        Detector2ExOrd(10000) As Integer
Dim SampleNo, MarkCount, MeanCount, BellMean, xval, yval, Photon_pairs, NoOfAngles
Dim CurrentMode$
Dim pi As Double

Private Sub ClrScreen_Click()
   Picture1.Cls
   Picture2.Cls
End Sub

Private Sub Form_Load()
   Active = 0
   HScroll1_Change
   HScroll1.Visible = True
   ActionFlag = 0
   All_Outcomes_Flag = False
   pi = 3.14159265358979
   Photon_pairs = HScroll4.Value - 1
   NoOfAngles = HScroll5.Value
   Text16.Text = NoOfAngles
   CurrentMode$ = "Statistical"

  'Set up cos2theta curve
   For i = 0 To Photon_pairs
      Cos2Theta(i) = (Cos((i * 90 / Photon_pairs) * pi / 180) ^ 2) * 4
   Next
   Text15.Text = HScroll4.Value
End Sub

Private Sub HScroll1_Change()
   SampleNo = HScroll1.Value
   Text1.Text = Format$(PhotonYield1(SampleNo), "0.####")
   Text2.Text = Format$(PhotonYield2(SampleNo), "0.####")
   Text4.Text = Ket1(SampleNo)
   Text5.Text = Ket2(SampleNo)
   Text6.Text = Format$(AngleDiff, "0.####")
```

```
    Text7.Text = Detector1Ord(SampleNo)
    Text8.Text = Detector1ExOrd(SampleNo)
    Text9.Text = Detector2Ord(SampleNo)
    Text10.Text = Detector2ExOrd(SampleNo)
    Text3.Text = Str$(SampleNo)
       If Active = 1 And Ket1(SampleNo) = 0 Then
       Text17.Text = Format$((PhotonAngle1(SampleNo) * 180 / pi), "##0.0#")
       Text18.Text = Format$((PhotonAngle2(SampleNo) * 180 / pi), "##0.0#")
    ElseIf Active = 1 Then
       Text18.Text = Format$((PhotonAngle1(SampleNo) * 180 / pi), "##0.0#")
       Text17.Text = Format$((PhotonAngle2(SampleNo) * 180 / pi), "##0.0#")
    End If
    If Active = 1 Then
       Text19.Text = Format$((PolarizerAngle1 * 180 / pi), "##0.0#")
       Text20.Text = Format$((PolarizerAngle2 * 180 / pi), "##0.0#")
    End If

End Sub

Private Sub HScroll2_Change()
    Text13.Text = HScroll2.Value / 10
End Sub

Private Sub HScroll3_Change()
    Text14.Text = HScroll3.Value / 10
End Sub

Private Sub HScroll4_Change()

    'Set number of photons-pairs in our set.
    Photon_pairs = (HScroll4.Value - 1)
    If Photon_pairs < 1 Then Photon_pairs = 1
    Text15.Text = Photon_pairs
      'Set up cos2theta curve
    For i = 0 To Photon_pairs
       Cos2Theta(i) = (Cos((i * 90 / Photon_pairs) * pi / 180) ^ 2) * 4
    Next

End Sub

Private Sub HScroll5_Change()
    ' Set number of angles to try (this also sets the number for average when angles are set
          manually)
    NoOfAngles = HScroll5.Value
    Text16.Text = NoOfAngles
End Sub
```

```
Private Sub MakeLight_Click()

   ' This subroutine genarates a set of photon-pairs, each pair orthoganal about axis of
         propagation.
   ' Photon source model
   Active = 1  'We show that we have a set of photon pairs so that gadgets can have values
   For SampleNo = 0 To Photon_pairs
      If Option1.Value = False Then       'If we select option to randomize orientation of the pair
         Randomize
         PhotonAngle1(SampleNo) = 360 * Rnd(1)
      Else                               'Otherwise, all pairs have 1 vertical and 1 horizontal photon
         PhotonAngle1(SampleNo) = 0      'and Photon 1 is horizontal
      End If
      PhotonAngle2(SampleNo) = PhotonAngle1(SampleNo) + 90
      temp = PhotonAngle2(SampleNo)
      If temp > 360 Then PhotonAngle2(SampleNo) = temp - 360
     ' convert to radians
      PhotonAngle1(SampleNo) = PhotonAngle1(SampleNo) * pi / 180  'Photon 1 of pair
      PhotonAngle2(SampleNo) = PhotonAngle2(SampleNo) * pi / 180  'Photon 2 of pair

     ' Randomize which ket goes to which analyzer
     ' If Ket1 is 0 then Photon 1 will go to Polarizer 1
     ' If Ket1 is 1 then Photon 1 will go to Polarizer 2
      Randomize
      Ket1(SampleNo) = Rnd(1)
      If Ket1(SampleNo) >= 0.5 Then
         Ket2(SampleNo) = 0
         Ket1(SampleNo) = 1
      Else
         Ket1(SampleNo) = 0
         Ket2(SampleNo) = 1
      End If
   Next
End Sub

Private Sub Picture1_MouseDown(Button As Integer, Shift As Integer, X As Single, Y As
         Single)
' This subroutine enables us to check the x- and y-values at any position in Picture 1

   Text11.Text = Format$(X, "##.####")
   Text12.Text = Format$(Y, "#.####")
End Sub

Private Sub Picture2_MouseDown(Button As Integer, Shift As Integer, X As Single, Y As
         Single)
```

```
' This subroutine enables us to check the x- and y-values at any position in Picture 2

    Text11.Text = Format$(X, "##.####")
    Text12.Text = Format$(Y, "##.####")
End Sub

Private Sub Plot_curves_Click()

   ' This subroutine represents the measurement device.
   ' It takes the output of the discriminators, and assumes perfect detection,
   ' and plots the the outcomes in several different ways.
    If Active = 0 Then
        MsgBox "Use Run1, Run2, or Photon Source and Measurement to generate Plot results"
        Exit Sub
    End If
   'Set up Picture 1
   'Plot range of output values scaled from 0 and 4, for mismatch between the two channels

    Picture1.ScaleMode = 0
    Picture1.Scale (-10, 4.5)-(100, -0.5)
    Picture1.AutoRedraw = True
    Picture1.Line (0, 4)-(90, 0), , B
    'Set up scales
    angle = 0
    While angle < 91
        Picture1.PSet (angle, 0)
        Picture1.Line -(angle, 0.1)
        Picture1.PSet (angle - 3, -0.2), QBColor(15)
        Picture1.Print Format$(angle, "0.0###")
        angle = angle + 15
    Wend
    fraction = 0
    While fraction < 4.1
        Picture1.PSet (0, fraction)
        Picture1.Line -(3, fraction)
        Picture1.PSet (-8, fraction + 0.1), QBColor(15)
        Picture1.Print Format$(fraction, "0.0###")
        fraction = fraction + 1
    Wend


' Set up Picture 2
' Plot the distribution of photons between analyzers, and the mean yield,
' as a function of the angle between polarizers (a reality check!)
    Picture2.ScaleMode = 0
    Picture2.Scale (-110, 1.1)-(100, -0.1)
```

```
Picture2.AutoRedraw = True
Picture2.Line (-90, 1)-(90, 0), , B
angle = -90
While angle < 91
   Picture2.PSet (angle, 0)
   Picture2.Line -(angle, 0.02)
   Picture2.PSet (angle - 3, -0.02), QBColor(15)
   Picture2.Print Format$(angle, "0.0###")
   angle = angle + 30
Wend
fraction = 0
While fraction < 4.1
   Picture2.PSet (-90, fraction)
   Picture2.Line -(-87, fraction)
   Picture2.PSet (-105, fraction + 0.02), QBColor(15)
   Picture2.Print Format$(fraction, "0.0###")
   fraction = fraction + 0.2
Wend

Yield1 = 0
Yield2 = 0
MeanYield = 0

' Generate Picture2 plot
'Plot average yield for the two channels as a function of polarizer angle difference

For j = 0 To Photon_pairs
   Yield1 = Yield1 + PhotonYield1(j)
   Yield2 = Yield2 + PhotonYield2(j)
   If Option8.Value = True Then        'If Analog option was set, we plot the analog version of
      the correlations
      xval = Abs(AngleDiff)
      While xval > 90
         xval = xval - 90
      Wend
      xval = xval * Sgn(AngleDiff)
      Picture2.Circle (xval, Abs(PhotonYield1(j) - PhotonYield2(j))), 1, QBColor(7) 'We plot
      the analog curve
   End If
Next

If Option8.Value = False Then
   ' We plot the mean values if we are not in the Analog option
   xval = Abs(AngleDiff)
   While xval > 90
      xval = xval - 90
```

```
    Wend
    xval = xval * Sgn(AngleDiff)
    MeanYield = (Yield1 + Yield2) / 2
    If Option8.Value = False Then
        Picture2.Circle (xval, Yield1 / (Photon_pairs + 1)), 1, QBColor(1)    'Blue
        Picture2.Circle (xval, Yield2 / (Photon_pairs + 1)), 1, QBColor(6)    'Yellow
        Picture2.Circle (xval, MeanYield / (Photon_pairs + 1)), 1, QBColor(0) 'Black
    End If
End If

' Generate Picture1 plot
' Count the number of times our detectors show the anti-correlation expected
' if polarizers are aligned.
' We are not measuring redundant counts, so we have to multiply by two to get
' the conventional correlation expected from 4 counts.
' In this version, we also keep counts for various Bell expectation values.

MeanBell1 = 0
MeanBell2 = 0
MeanBell3 = 0
MarkCount = 0

Root2 = Sqr(2)
For i = 0 To Photon_pairs
    'Plot correlation of outcomes in the two detectors
    If Detector1Ord(i) <> Detector2Ord(i) Then MarkCount = MarkCount + 1
    If Detector1ExOrd(i) <> Detector2ExOrd(i) Then MarkCount = MarkCount + 1
    If All_Outcomes_Flag = True Then    'The Outcomes_Flag tells us to plot the Bell
        expectations

        ' Q is value of Detector1Ord, R is Detector1ExOrd, S is Detector2Ord, T is
        Detector2ExOrd
        ' Bell local (BellCount1 --> MeanBell1 or MeanBell3) is QS + RS + RT - QT

        QS = Detector1Ord(i) * Detector2Ord(i)
        RS = Detector1ExOrd(i) * Detector2Ord(i)
        RT = Detector1ExOrd(i) * Detector2ExOrd(i)
        QT = Detector1Ord(i) * Detector2ExOrd(i)


        BellCount1(i) = QS + RS + RT - QT   'This represents Bell's local realistic expectation
        BellCount2(i) = (QS + RS + RT - QT) * Root2    'This represents Bell's quantum
        expectation, equivalent to
        '(QS * 1 / Root2 + RS * 1 / Root2 + RT * 1 / Root2 - QT * 1 / Root2) * 2,
        ' which is numerically equivalent to the values from McHarris' QM treatment (see
        Footnote 3 in text)
```

```
        MeanBell3 = MeanBell3 + BellCount1(i)
        ' This coincidence count is equivalent to MarkCount x 2, the correlation outcome above,
        but with sign reversed

        ' The values from (QS + RS + RT - QT) are +/- 2. To get the standard mean expectation
        values, we have to sum the absolute values.

        MeanBell1 = MeanBell1 + Abs(BellCount1(i))  'See last remark; - BellCount1 value can
        be + or - 2)
        MeanBell2 = MeanBell2 + Abs(BellCount2(i))  'See last remark; - BellCount2 can be +
        or - 2*Root2
      End If
    Next i

    If All_Outcomes_Flag = True Then
      MeanBell1 = MeanBell1 / (Photon_pairs + 1)
      MeanBell3 = MeanBell3 / (Photon_pairs + 1)
      MeanBell2 = MeanBell2 / (Photon_pairs + 1)
      BellMean = BellMean + MeanBell3
    End If

      xval = Abs(AngleDiff)

      While xval > 90
        xval = xval - 90
      Wend
   ' To avoid redudancy, since we assume our discriminators are perfect,
   ' we are only scoring two correlation values, instead of 4, so we multiply the outcomes by 2.
   ' If our photons come in at random orientations (or of we randomize polarizers),
   ' we expect the mean value to range between 1 and 3, and have to normalize
   ' by multipying by 4, and offsetting by -2.

      If Option2.Value = True And CurrentMode$ = "Statistical" Then
        yval = (MarkCount * 4 / Photon_pairs) - 2      'Normalize count to range 0-4
      ElseIf Option1.Value = True And CurrentMode$ = "Statistical" And Option6.Value = True
        Then
        yval = (MarkCount * 4 / Photon_pairs) - 2      'Normalize count to range 0-4
      Else
        yval = MarkCount * 2 / (Photon_pairs)          'Normalize count to range 0-4
      End If
      MeanCount = MeanCount + yval
      If Option7.Value = True And RunNo = NoOfAngles Then     'If we have a fixed angle, we
        can improve our S/N by averaging
        yval = MeanCount / NoOfAngles
        Picture1.Circle (xval, yval), 0.6, QBColor(0)
```

```
        ElseIf Option7.Value = False Then
            Picture1.Circle (xval, yval), 0.4, QBColor(0)
        End If

'       Picture1.Circle (xval, yval), 0.4, QBColor(0)
        If All_Outcomes_Flag = True Then
            If Option7.Value = True And RunNo = NoOfAngles Then     'If we have a fixed angle,
            we can improve our S/N by averaging
                yval = (BellMean / NoOfAngles) + 2
                Picture1.Circle (xval, yval), 0.4, QBColor(8)
            ElseIf Option7.Value = False Then
                Picture1.Circle (xval, MeanBell3 + 2), 0.4, QBColor(8)  'mean of (QS + RS + RT -
            QT)
            End If
            Picture1.Circle (xval, (MeanBell1)), 0.4, QBColor(5)    'mean of Abs(QS + RS + RT -
            QT)
            Picture1.Circle (xval, MeanBell2), 0.4, QBColor(12)
        End If

        If ActionFlag = 1 Then
            Mark(RunNo, 0) = xval      'Store data values
            Mark(RunNo, 1) = yval
        End If

    ' Plot theoretical curve expected for entangled photons
        For i = 0 To Photon_pairs
            xval = i * 90 / (Photon_pairs) 'Normalize to degrees
            Picture1.PSet (xval, Cos2Theta(i)), QBColor(4)
        Next
    ' Draw diagonal expected from Bell local realistic model
        Picture1.Line (0, 4)-(90, 0)

End Sub

Private Sub Quitit_Click()
    End
End Sub

Private Sub Run_it_Click()
    If Option8.Value = False Then
        If Option3.Value = True Then
            CurrentMode$ = "Binary"
        ElseIf Option4.Value = True Then
            CurrentMode$ = "Statistical"
        End If
    End If
```

```
    MeanCount = 0
    BellMean = 0

    'Run number of experiments from No. Angles slider
    ActionFlag = 1
    For RunNo = 1 To NoOfAngles
        MakeLight_Click
        Set_polarizers_Click
        Plot_curves_Click
    Next
    ActionFlag = 0
End Sub

Private Sub Run2_it_Click()
    If Option8.Value = False Then
        If Option3.Value = True Then
            CurrentMode$ = "Binary"
        ElseIf Option4.Value = True Then
            CurrentMode$ = "Statistical"
        End If
    End If

    MeanCount = 0
    BellMean = 0

    'Run number of experiments from No. Angles slider
    ActionFlag = 1
    MakeLight_Click
    For RunNo = 1 To NoOfAngles
        Set_polarizers_Click
        Plot_curves_Click
    Next
    ActionFlag = 0
End Sub

Private Sub Save_As_Click()
    CommonDialog1.ShowSave
    FileName$ = CommonDialog1.FileName
    Open FileName$ For Output As #1
    For i = 0 To 99
        Print #1, Mark(i, 0), Mark(i, 1)
    Next
    Close #1

End Sub
```

```
Private Sub Set_polarizers_Click()

' Set the angles for our polarizers, and match to a ket,
' then calculate the probable yield using Malus Law

' Polarizer 1
   If Option6.Value = True Then    'We randomize both polarizers
      Randomize                    'Set polarizer 1 at random between -45 and 45 degrees
      Polarizer1 = 90 * (Rnd(1) - 0.5)
   ElseIf Option5.Value = True Then
      Polarizer1 = 0               'Set polarizer one at fixed angle of 0
   ElseIf Option7.Value = True Then
      Polarizer1 = HScroll2.Value / 10
   End If
      ' convert to radians
      PolarizerAngle1 = Polarizer1 * pi / 180
' Polarizer 2
   If Option7.Value = True Then
      Polarizer2 = HScroll3.Value / 10
   Else
      ' We have to randomize the angle between the polarizers to generate all values.
      Randomize
     'Set polarizer 2 at random between -90 and 90 degrees with respect to polarizer 1
      Polarizer2 = Polarizer1 + 180 * (Rnd(1) - 0.5)
   End If
      ' convert to radians
      PolarizerAngle2 = Polarizer2 * pi / 180

      AngleDiff = (PolarizerAngle1 - PolarizerAngle2) * 180 / pi
      Text6.Text = Format$(AngleDiff, "0.####")
      Text6.Refresh



' Now sample the 1000 photons at random angles
   For SampleNo = 0 To Photon_pairs

'If Ket1 is 0 then Photon 1 (nominally H) will go to Polarizer 1
'If Ket1 is 1 then Photon 1 will go to Polarizer 2
   If Ket1(SampleNo) = 0 Then  ' this is Photon 1 of pair
         PhotonYield1(SampleNo) = (Cos(PolarizerAngle1 - PhotonAngle1(SampleNo))) ^ 2
      Else
         PhotonYield1(SampleNo) = (Cos(PolarizerAngle1 - PhotonAngle2(SampleNo))) ^ 2
      End If
```

```
'This is where we implement the discriminator properties
    ' Normalize yield to conform to Law of Malus, and comply with quantized distribution
    If CurrentMode$ = "Statistical" Then
        Randomize
        testval = Rnd(1)    'Use a random value to test; Malus' law is a statistical law
    Else
        testval = 0.5       'We have to conserve our photons, so we distribute them according to a
            generic 50-50 (binary) probability
    End If
        If PhotonYield1(SampleNo) > testval Then
            Detector1Ord(SampleNo) = 1
            Detector1ExOrd(SampleNo) = -1
    ' But assume that if not in Ordinary ray, will appear in Extraordinary ray
        Else
            Detector1Ord(SampleNo) = -1
            Detector1ExOrd(SampleNo) = 1
       End If


'If Ket2 is 0 then Photon 1 will go to Polarizer 2
'If Ket2 is 1 then Photon 1 will go to Polarizer 1
        If Ket2(SampleNo) = 0 Then  ' Photon 2 of pair
            PhotonYield2(SampleNo) = (Cos(PolarizerAngle2 - PhotonAngle1(SampleNo))) ^ 2
        Else
            PhotonYield2(SampleNo) = (Cos(PolarizerAngle2 - PhotonAngle2(SampleNo))) ^ 2
        End If
    ' Normalize yield to conform to Law of Malus
    If CurrentMode$ = "Statistical" Then
        Randomize
        testval = Rnd(1)    'Use a random value to test
    Else
        testval = 0.5
    End If
        If PhotonYield2(SampleNo) > testval Then
            Detector2Ord(SampleNo) = 1
            Detector2ExOrd(SampleNo) = -1
    ' But assume that if not in Ordinary ray, will appear in Extraordinary ray
         Else
            Detector2Ord(SampleNo) = -1
            Detector2ExOrd(SampleNo) = 1
       End If
    Next

End Sub

Private Sub Toggle_Flag_Click()
```

```
    If All_Outcomes_Flag = True Then
        All_Outcomes_Flag = False
    ElseIf All_Outcomes_Flag = False Then
        All_Outcomes_Flag = True
    End If
End Sub
```