Repeated Measures and Interventions
Longitudinal Data
(aka Repeated Measures)

- The same observational unit (plot, individual, etc) is often measured repeatedly over time
- Usually have **many** such observation units
- Observations on the same unit over time are not independent
Alternatives for Repeated Measures

• Random effects
  – By time: Assumes all observational units move up or down in sync
    • probably won't solve lack of independence
  – By unit: Assumes a unit is offset from “average” by some constant amount

• Autoregressive: AR(1)
  – Assumes each unit is similar from one time step to the next but not that units are synchronized

• With short t.s. almost impossible to distinguish AR vs individual effects
Example: Growth

• Consider a population of individuals censused 3 times whose growth is a function of $X$

$$
\mu_{i,t} = \beta_0 + \beta_1 X_{i,t}
$$

• If we assume a common variance and autocor.

$$
\tilde{g}_i \sim N_3(\tilde{\mu}_i, \Sigma)
$$

$$
\Sigma = \frac{\sigma^2}{1-\rho^2} \begin{pmatrix}
1 & \rho & \rho^2 \\
\rho & 1 & \rho \\
\rho^2 & \rho & 1
\end{pmatrix}
$$
Likelihood Approach

```r
library(mvtnorm)
H <- matrix(c(0,1,2,1,0,1,2,1,0),3,3)

lnlik <- function(theta){
  beta <- theta[1:2]
  sigma <- theta[3]
  rho <- theta[4]
  SIGMA <- sigma/(1-rho^2)*rho^H

  L = 0
  for(i in 1:n){
    L = L - dmvnorm(g[i,],beta[1]+beta[2]*X[i,],SIGMA,log=TRUE)
  }

  return(L)
}
```
rho = 0.654

Residual Error
AR(1) = 1.5
LM = 2.1
Bayesian Approach

model{
  beta[1] ~ dnorm(0,0.01)
  beta[2] ~ dnorm(0,0.01)
  sigma ~ dgamma(0.01,0.01)
  rho ~ dunif(-1,1)
  SIGMA <- inverse(1/sigma/(1-rho^2)*rho^H)

  for(i in 1:n){
    mu[i,] <- beta[1]+beta[2]*X[i,]
    g[i,] ~ dmnorm(mu[i,],SIGMA)
  }
}
State Space approach

• Bayesian version of AR(1) repeated measures did not separate process and measurement uncertainty

• Easy to extend the State Space model to the repeated measures context with a common rho

\[ g_{i,t} \sim N(\beta_0 + \beta_1 X_{i,t} + \rho g_{i,t-1}, \sigma^2) \]

\[ g_{i,t}^{(o)} \sim N(g_{i,t}, \tau^2) \]
Generalization of Repeated Measures

- Can accommodate mixed models / random effects (though some would be redundant)
- Can accommodate nonlinear models
- Covariance matrix ONLY works with NORMAL PDF
- State Space version can work with most any PDF because we separate the data model (not autocorrelated) from the process error (autocorrelated)
Missing Data Covariance Matrix

- Drops missing ROWS and COLUMNS from matrix

\[
\begin{bmatrix}
1 & \rho & \rho^3 & \rho^4 & \rho^5 \\
\rho & 1 & \rho^2 & \rho^3 & \rho^4 \\
\rho^3 & \rho^2 & 1 & \rho & \rho^2 \\
\rho^4 & \rho^3 & \rho & 1 & \rho \\
\rho^5 & \rho^4 & \rho^2 & \rho & 1
\end{bmatrix}
\]

Missing 3\textsuperscript{rd} observation so drop both the third row and third column from the full matrix

\[
\begin{bmatrix}
X1 & X2 & X4 & X5 & X6
\end{bmatrix}
\]
Hierarchical AR models

- Thus far have focused on autocorrelation in the STATE variables as part of the process model

- In hierarchical models, can also model autocorrelation in the hierarchical parameters
  - Time-varying model parameters

\[
\tilde{\alpha} \sim N(0, \tau^2)
\]

\[
\hat{\alpha} \sim N(0, \Sigma)
\]
Fire in the Florida Everglades

ARMA(2,2) with temporally varying parameters

Beckage and Platt 2003
Intervention Analysis

- Treatment effects in TIME
  - Pretreatment data establishes unit differences
- Hypotheses are usually that one or more parameters changed with treatment
- Alternate model (NULL) is no change in parameters with treatment
- Time can be modeled as
  - Covariate (explicit)
  - Time varying treatment (implicit)
  - Autocorrelation
FIGURE 9.21. Four examples of ways in which an intervention might occur. The trend in the solid line "Treatment A" might not match that of the response to the treatment. For example, a step change in a treatment $x$ (a) might elicit a trajectory of change in a response variable $y$. 
Change point / threshold detection models

- Like Intervention models but the point in time where the change in parameters occurs is itself an unknown that needs to be estimated
- Challenging for likelihood models because the likelihood has a discontinuity