**Supplementary Information**

**Section 1. The BCHSH consensus treatment**

The early experiments\(^1\)\(^2\)\(^3\)\(^4\) considered photons, with correlated orientation designated by \(H\) (horizontal) or \(V\) (vertical), for pairs in Bell-states \(HV/VH\) or \(HH/VV\). Coincidence of detection at two stations was tested at four settings of the polarizers in a setup such as that in Fig. 1. With \(\alpha\) or \(\alpha'\) and \(\beta\) or \(\beta'\) as polarizer settings respectively at stations 1 and 2, the approach considers possible coincidences at settings \(\alpha, \beta; \alpha, \beta'; \alpha', \beta\); and \(\alpha', \beta'\), chosen to discriminate between outcome expectations. These come from coincidences in pairwise measurements at separate stations, \((E_{a,b}, \text{etc.})^3\), and then give outcomes summed through:

\[
S_{BCHSH} = E_{a,\beta} + E_{a,\beta'} + E_{a',\beta} - E_{a',\beta'} \quad (eq. 1a),
\]
where \(E\) values reflect normalized coincidence yields at the four settings. With conventional scoring, at the elemental level, a value of +1 was assigned to a photon detected in the ordinary ray, and of -1 if in the extraordinary ray, and each of the four \(E_{a,b}\), etc. terms can have a value \(\pm 1\). The sum \(S_{BCHSH}\) is then constrained to the range \(\pm 2\) so that \(S_{BCHSH}\) is always \(\leq 2\).

\[
-2 \leq S_{BCHSH} = E_{a,\beta} + E_{a,\beta'} + E_{a',\beta} - E_{a',\beta'} \leq 2 \quad (eq. 1b)
\]

*Orthodox QM expectations.* In the Bell’s treatment for electrons, although the vectorial properties of each pair are indeterminate, the quantum entities retain the correlation resulting from conservation of angular momentum in the transition generating them, represented in effect by orientation of spin states in the wavefunction. Evolution to a measurement context resolves quantum uncertainties on actualization in real vectorial states. Bell’s\(^5\) expression for electrons predicts the outcome through:

\[
(\hat{\sigma}_1 \cdot \hat{\alpha} \hat{\sigma}_2 \cdot \hat{\beta}) = -\hat{\alpha} \cdot \hat{\beta} = -\cos \sigma \quad (eq. 2),
\]
which involves a conventional operation of the Pauli matrices, \(\sigma_1\) and \(\sigma_2\) on the field vectors, \(\alpha\) and \(\beta\), of the Stern-Gerlach magnets. The partners in a pair are correlated through dichotomic spin quantum numbers, \(s \pm \frac{1}{2}\). In the wave function, possible spin states are shown by their phase difference, related to the spin quantum numbers by \(\pi/2s\), with the 180\(^\circ\) difference represented, for example, by \((\uparrow, \downarrow)\). Possible spin combinations of the electron pair are represented in the matrices. The matrix operations then explore potentialities through projections between electron and discriminator vectors (eq. 2), resolved to give observables. The justification for these states is discussed in detail in the text. Each measurement detects one of the two states. With allowance for spin quantum number difference, \(s \pm 1\), the same treatment applies to photon pairs, with phase difference 90\(^\circ\), represented by \(H\) and \(V\) spin states.
Although eq. 1a is generally treated as appropriate to OL models, it is obvious, both from the simulation, and because \( \cos \sigma (\text{or} \cos^2 \sigma) \) varies over \( \pm 1 \), defining the same range, that with photons it must also be appropriate to the NL treatment. For photon pairs in superposition, real properties cannot be assigned. A wave equations can represent the different states by using \( H \) or \( V \) (for example, Bell-states \( HV/VH \) or \( HH/VV \), with wavefunction \( |\psi\rangle = \frac{(|H_AV_B\rangle + |V_AH_B\rangle)}{\sqrt{2}} \) for the \( HV/VH \) case). However, \( H \) and \( V \) do not indicate vectors; the spin designators (phase differences) attain vectorial significance only in the context of a reference frame. The only vectors assigned before measurement are those for the polarizers, with the fixed polarizer determining the reference frame. At any orientation of the photon frame (including stochastic), the matrix operations align an \( H \) photon with the fixed polarizer as a common reference for all frames\(^3\) (or a \( V \) photon orthogonal), and the second photon is then actualized in the complementary orientation (determined by the Bell state) at the variable polarizer. The justification for this alignment is by no means obvious, but, as discussed at length in the text, involves attribution to the spin states of propensities. The projections between photon and polarizer vectors are then the same as in the vOL treatment when the frames are aligned. In the NL treatment the projections represent potentialities in the complex plane which can be resolved on taking products of the two roots to give observables, - the same Malus’ law probabilities as from the vOL model, - yielding for example, \( E_{\alpha,\beta}(\text{NL}) = \cos^2 \sigma - \sin^2 \sigma = \cos 2\sigma \), etc., with \( \sigma = \beta - \alpha \), the angle difference between the polarizers (cf.\(^1-5\)). The predicted outcome when \( \sigma \) is varied is a \( 2\cos^2 \sigma \) curve, conventionally scaled in the range \( \pm 1 \). To yield the set at four canonical angle differences, values for \( S_{\text{NL}} \) (eq. 1a) are calculated at two different settings of the fixed polarizer. Then, \( S_{\text{NL}} \) is the sum of \( E_{\alpha,\beta}(\text{NL}) \), etc. terms taken from two such curves, one at \( \alpha \), the other at \( \alpha' \), for which the coincidences at settings \( \beta \) and \( \beta' \) of the variable polarizer are determined. Because in the NL treatment the photon frame always becomes aligned with the fixed polarizer, and \( \sigma \) is referred to that, the same \( 2\cos^2 \sigma \) curve is expected with any setting of the fixed polarizer. Summation of two curves then gives the characteristic \( S_{\text{NL}} \) outcome scaled to \( 4\cos^2 \sigma \), conventionally in the range \( \pm 2 \). Using canonical values \( \alpha = 45^\circ \), \( \alpha' = 0 \), \( \beta = 22.5^\circ \), and \( \beta' = 67.5^\circ \), so that \( \sigma \) has values of \( \pm 22.5^\circ \) or \( \pm 67.5^\circ \) (\( \cos 2\sigma = \pm 0.707 \)), substitution in eq. 1a then gives \( S_{\text{NL}} = 2.83 \). Since the curve at any setting of the fixed polarizer is the same, the \( 2\cos^2 \sigma \) curves are commonly presented as predicting the full-visibility NL outcome, though they have to be rescaled to four to give \( S_{\text{NL}} \).

\( \text{OL expectations.} \) There are three distinct derivations of the OL limit of \( \leq 2 \).
(i) The BCHSH treatments anticipated experiments with photon pairs from atomic cascades, and the property of interest, in this case, \( \lambda \), defining the polarization vector of each photon, was included in analysis of local probabilities. A clear exposition is the treatment by Clauser and Horne. Following this, the local mean yield at station 1 is derived by integration of elemental yields, \( p_1(\lambda, \alpha) \), by

\[
p_1(\lambda, \alpha) = \int \rho(\lambda)p_1(\lambda, \alpha)d\lambda \quad \text{(eq. 3a)}
\]

This gives the mean from a stochastic population measured at polarizer setting \( \alpha \), with equivalent terms for other polarizer settings. The function \( \rho(\lambda) \) is a normalized probability distribution for \( \lambda \). For a stochastic state, the population would be isotropic in the plane of measurement, so that on integration, contributions from photon vectors would cancel through \( \int \rho(\lambda)d\lambda = 1 \) (eq. 3b). From this, the outcome of eq. 3a is the same at all polarizer orientations (remember those experiments with polaroid glasses from your childhood), and no particular values for \( \lambda \) can be determined from a local measurement (epistemologically, \( \lambda \) is a “hidden variable”).

Calculation of expectations for coincidence counts is sketched out for settings \( \alpha, \beta \) as follows:

\[
E_{\alpha, \beta(\text{LR})} = \int \rho(\lambda)p_{12}(\lambda, \alpha, \beta)d\lambda = \int \rho(\lambda)p_{12}(\lambda, \alpha, \beta)d\lambda = p_{1,2}(\alpha, \beta) \quad \text{(eq. 4)}
\]

In the first term on the right (RHT1), the probability of coincidence in pairwise measurements is calculated from the product of separate probabilities, and the mean from a population is given by the integral. This the standard analysis. However, in the second equation (RHT2), all the variables (\( \lambda, \alpha, \beta \)) were included in a single expression. This would then lead on integration to the cancellation of terms in \( \lambda \) implicit in eq. 3b. The value for \( E_{\alpha, \beta(\text{LR})} \) would then have to be independent of \( \lambda \), but depend on \( \sigma \), the difference between polarizer settings; in effect the photon vectors have been excluded as determining variables.

Bell dealt with the loss of the information on photon vectors by basing his local treatment on scalar dichotomic correlations (the sign of the spin, \( \pm 1 \)). Expectation values were then given by:

\[
E_{\alpha, \beta(\text{OL})} = p_{1,2}(\alpha, \beta) = -1 + 2\sigma/\pi, \quad \text{(eq. 5 etc., see \textit{\cite{9}} for a detailed derivation)}.
\]

As in the NL approach, the critical variable was the difference between polarizer orientations, \( \sigma \), but now giving a linear zigzag curve. The four values giving \( S_{\text{OL}} \) (eq. 1a) were then taken from a zigzag constrained to the range \( \pm 2 \), assumed in all early accounts to be characteristic of OL expectations. With measurement at the four canonical settings, the value for \( S_{\text{OL}} \) expected from the zigzag was 2, giving the OL limit.

(ii) Shimony, Clauser and colleagues (CHSH) derived the same limit from a different method, - consideration of counts of elemental pairwise correlations. This approach has been widely
adopted\textsuperscript{4,8,15-17}. Correlations can be determined only if a photon is detected at each station. With a polarization analyzer, a photon is either in the ordinary or extraordinary ray. In the consensus formulation, a value of +1 was assigned to a photon detected in the ordinary ray, and of -1 if in the extraordinary ray. Then, at settings $\alpha, \beta; \alpha, \beta'$; $\alpha', \beta$; and $\alpha', \beta'$, each coincidence would score either 1 or -1, and the four pairwise comparisons could yield no more than 4 coincidences, in the range $\pm 2$ (eq. 1b).

Since means from measurement on a population cannot exceed the elemental values, this determines a limit for $S_{OL} \leq 2$.

The limits in (i) and (ii) are both derived from eq. 1a and, although derived through different methods, were taken to be equivalent in the early literature.

(iii) Bell\textsuperscript{5} also hinted at a third distinction, “…consider the result of a modified theory…in which the pure singlet state is replaced in the course of time by an isotropic mixture of product states…”, for which Bell suggested a correlation function, $(-\frac{1}{\sqrt{3}} a\cdot\beta)$, which would give a sinusoidal curve of reduced amplitude (and also $< 2$) compared to NL models. In contrast to the first two, this third inequality is based on vectorial analysis, and is discussed at greater length in Part B of the main text.

\textit{Section 2. Epistemological and ontological perspectives, and the uncertainty principle}

\textit{i) Where in the QM treatment does actualization occur?}

For Bell in 1987, a problem of principle remained “that of locating precisely the boundary between what must be described by wavy quantum states on the one hand, and in Bohr’s ‘classical terms’ on the other” (in Preface\textsuperscript{10}). Surely this boundary must reflect where in the process discrete states are actualized, - what Bell had earlier called “…the notoriously vague “reduction of the wave packet”…”\textsuperscript{8}.

In state preparation, the photons have their vector properties massaged by passage through refractive components. These processes determine at which detector the photon will generate a signal. The observer comes into the picture only in \textit{interpretation} of the outcome, - in mapping the causal chain between generation of the photon pair and the response of detectors. The observer is of no consequence to the physical outcome; that is determined by state preparation protocols before the photon arrives at a detector. In illustration of this, a scenario has recently been reported, in which the entangled source (a polarization Sagnac interferometer, see Fig. SI\_1A) and all state preparation optics were housed in a satellite, and polarized separate
populations of “entangled” pairs were beamed to ground stations separated by 1200 km for analysis\textsuperscript{18}. Does that distance make any difference? No, - when the photons leave the satellite, they are already destined to light up a particular detector by what happened during state preparation. Would changing any distance matter? In this experiment, the distances between the satellite and each station varied widely, but GPS timing and additional information provided by ancillary beams was used to implement compensation in alignment of counts to score coincidences. This opens a set of interesting questions as to where actualization occurred in these experiments, and the more general question of where it must occur.

When superposition is invoked, real properties of the photons would be needed at the polarizers to justify use of Malus’ law. In fact, since both properties of the photon (frequency, and the action vector) are engaged in refraction, the photons must have real and local properties at any refractive event; this means that a photon must be actualized at the first refractive element encountered, and if entangled, this must lead to simultaneous actualization of its partner. All subsequent processes would then be determinate. Any refractive interrogation earlier than the polarizer then undermines the NL scenario, or at least the version commonly cited.

\textit{ii) ‘Tensions’ with relativity.}

Under a consensus from the 1980s following the strong proof that quantum field theory cannot provide faster-than-light communication\textsuperscript{19}, problems involving such communication were sometimes dismissed as of no significance because comparisons needed to discern correlations were all made after the event through subluminal exchanges\textsuperscript{19-21}. Surely this was wishful thinking. Bob and Alice at space-like separated stations record polarizer orientation, time of measurement, and detector response locally. Although correlations between stations cannot be evaluated without exchange of this information, no correlations appropriate to the outcome claimed would have been seen \textit{unless} the data recorded was appropriate. This would have been the case only if photon and polarizer orientations had \textit{particular} values \textit{at the time of measurement}. The correlation of spins by phase difference shown in the wavefunction is not enough. The alignment achieved is a mathematical trick arising from the use of propensities in the matrix operations\textsuperscript{3}. The outcome depends on different behaviors at the two polarizers. Starting with an indeterminate state, the process of alignment necessary for any particular pair requires superluminal transfer to station 2 of \textit{specific information} about the photon actualized on measurement \textit{at that instant} at station 1.
Was that photon $H$ or $V$? What was the polarizer 1 orientation? Without that information the scheme won’t work.

A similar rationale for dismissing the conflict is the claim that transfer of information is not subject to relativistic constraints$^{3,21-24}$. Shimony discusses several variants, including somewhat opaque explanations from Zeilinger$^{24}$ and Peres$^{21}$, but the following general case suggested by Jaynes$^{25}$ is clearer and may be equivalent: “The measurement at $A$ at time $t$ does not change the real physical situation at $B$; but it changes our state of knowledge about that situation, and therefore it changes the predictions we are able to make about $B$ at some time $t_0$. Since this is a matter of logic rather than physical causation, there is no action at a distance and no difficulty with relativity”. However, if “…the real physical situation at $B$…” is unchanged, it must be represented by real properties. To avoid relativistic problems, these properties would have to be intrinsic to the state. Knowledge accrued “…on measurement at $A$…” then reflects an understanding from Einstein’s rather than Bohr’s perspective. On the other hand, in the indeterminate superposition “…the real physical situation…” is, by predicate, unknowable. If so, as discussed in the main text (B, Sections 6, 7), the specific vectorial values applied in projections to give the outcome claimed would have to depend on information transferred superluminally. A way around this is to invoke propensities as causal agents, but that could only work if real vectors pre-aligned with the polarizer frame were conferred on the spin states. I’ve called this process a Maxwellian demon; it has no justification in physics, and also requires an ontic dichotomy of spin states, itself unjustified.

An alternative explanation to be considered is that information can be transmitted superluminally in a communication. This introduces the question of the physical status of different components of the communication process. In a communication, information is encoded at an emitter station through changes in some physical carrier, transferred to a receiver location by the carrier, and decoded there to reveal a message. As Shannon$^{26}$ noted, the semantic component of a communication (the meaning of the message) lies outside these ‘engineering’ considerations. As discussed elsewhere$^{27}$, by this definition the meaning makes no thermodynamic contribution, - it is insubstantial. It might therefore be suggested that transfer of information is not subject to relativistic constraints. However, this ignores the need for a physical apparatus. Although “meaning” might be insubstantial, the message itself must be encoded by modification of a physical carrier and decoded by a suitable acceptor function. This requires a causal chain of physical interactions...
of components, including translational machineries at both ends, etc. It is only in metaphysics that communication can occur without such a physical framework, and recognition of this constraint then provides a useful demarcation between science and metaphysics. The suggestion that meaning can be transmitted without physical agency then comes up against the same very strong case against superluminal communication; treatments incorporating such thinking (all conventional NL treatments) must be excluded from the scientific side on the basis of the closure of communication loopholes.

As a counter argument, I should note that in some conversations, it has been pointed out to me that if gravity can act instantaneously over space-like distance through field effects, invocation of Maxwell’s equations in quantum field theories then opens the possibility that properties of the wave might be instantaneously available at space-like separated stations. The artificiality of this argument is addressed in Section 4.

**iii) Hidden-variables**

It was the famous refutation of von Neumann’s proof of the impossibility of hidden variables in quantum theory (previously addressed by Grete Hermann, though her work had been ignored and forgotten) that led Bell into the entanglement issue. His use of the term ‘hidden variables’ in the context of the EPR argument was later challenged by Jammer because Einstein never used the term. In his rebuttal, Bell suggested that Einstein’s claim that the Copenhagen treatment was incomplete was tantamount to a claim that QM treatments required hidden variables. However, that is not the context in which he used the term later in the entanglement debate. Bell found that von Neumann’s argument still held in the context of the standard QM model, so that the validity of the Copenhagen interpretation was not, in his mind, in question. Einstein’s incompleteness argument then seemed inconsequential in that context. However, in evolution of Bell’s local realistic (LR) argument, the apparent inaccessibility of vectorial information under LR constraints suggested that ‘hidden variables’ might be considered as possible carriers of information that could be used in the measurement context to provide information allowing “conspiracies” leading to the experimental outcome otherwise demanding the NL perspective. This audacious switcheroo paradoxically shifted the question of hidden variables from the QM scenario, where vectorial properties from the generating transition are discarded in the superposition
but then re-invented on actualization, to the OL treatment, where vectorial properties were con-
sidered to be inaccessible and therefore discarded, seemingly then requiring hidden variables and
conspiracies to explain the experimental outcome. Experimental exclusion of superluminal trans-
fer then locked consideration of hidden variables out of the debate, thereby excluding all OL
models, including Einstein’s.

It will be obvious that my naïve simulation throws a spanner into this logic. It demon-
strates that, in a model strictly compliant with local realistic constraints, application of EPR’s
primary premise (that quantum entities carry intrinsic properties) can generate outcomes ob-
served experimentally, simply by counting coincidences. The model works without recourse to
any mathematical sophistication. This result alone should demand a re-examination of the con-
ventional edifice. The simulation strips away the mathematical masquerade of the NL model, and
shows that the only trick needed to generate the special rotational invariance is the alignment of
frames. However, when that is implemented in the matrix operations, its artificiality is demon-
strated by the consequent conflicts with second law and relativistic constraints. Although I have
called it a demon, the solution implicit in invocation of propensities with real effects could be
seen as introduction of a grotesque “hidden variable”, calling into question both von Neumann’s
claim, and Bell’s acquiescence.

I am a great admirer of Bell, - he applied his ideas with fearless logic, and was certainly
not uncritical of received opinion in the entanglement debate. But his elimination of Einstein’s
‘element of reality’ stranded him with an OL model stripped of the one property essential in gen-
eration of the sinusoidal outcome. It was just this artificiality, - that superposition stripped the
vectors from the QM treatment, - that Einstein had regarded as indicating that the QM treatment
was “incomplete”. As discussed at length in the main text, even with the matrix operations, the
NL treatment cannot provide the information needed except through superluminal transfer. Of
the three mechanisms discussed (the NL, OL and \(v\)OL (Einstein’s) models), only the latter is nat-
ural. However, because of locality constraints, it cannot predict the NL outcome claimed.

iv) ‘Tensions’ with the second law.

Two entropy-related features crop up in the entanglement debate and have to be consid-
ered separately.
a) **Re-ordering of the state.** Perhaps the most compelling case for NL expectations comes from the early experiments with atomic cascade sources, in which a full-visibility outcome is claimed starting with a stochastic source. With correlated LR photon pairs generated on excitation of an atomic cascade in stochastic orientation, rotational invariance on measurement is expected, but with the reduced amplitude for the outcome curve seen in the LR2 curves. Dissection of this state (see Fig. 6 in the main text) brings out features not obvious from standard treatments. The photons partition according to their angle difference with the polarizer. Since a stochastic photon population has every orientation, the yields at any polarizer setting will partition in a normal distribution about the polarizer angle, $\theta$, with a mean yield in each ray of 0.5 (the singles count). The mean yields from ordinary and extraordinary rays will sum to 1 at each station (2 overall). The correlated photons of a pair from a cascade source have the same $\lambda$, but will fall on polarizers differing in orientation, $\theta_1$, $\theta_2$, with different probabilities of transmission, $\cos^2(\theta_1 - \lambda)$, $\cos^2(\theta_2 - \lambda)$, and coincidence will reflect $\cos^2\sigma$, with $\sigma = (\theta_1 - \theta_2)$. With detectors at ordinary and extraordinary rays at both stations, and two pair-configurations ($HH$, and $VV$ in this case), eight probabilities contribute to the coincidence count at any setting. The green curves (Fig. 6 in the main text) were generated by plotting this function for the orientation of a random photon pair from the stochastic population examined at each setting of the variable polarizer (the value for $\lambda$ of the initial pair). The envelope sampled by the green curves shows graphically the range of possible contributions to the coincidence count. These are all dependent on angle difference; every photon pair contributes to the LR2 curve, but coincidences would be detected only inside the envelope. The half-visibility of the LR2 curve represents the count remaining after partial cancellations. While photon pairs at all values for $\lambda$ contribute to the envelope (and hence the coincidences), they also, as alignments depart from the QM ideal, contribute to the cancellations. The LR2 curve bisects the envelope, so both curve and envelope follow the Malus’ law dependence on angle difference between the polarizers, with the envelope occupying half the scale, as expected from the above. Any treatment recognizing a disordered source would necessarily show these effects. With cascade sources (with the photon population stochastic), or if PDC sources are misaligned, the full-amplitude rotational invariance claimed would require a re-ordering to align each photon pair. Unless a work term and a mechanism for its application can be identified, the claim of full-visibility is untenable.
b) The entropy associated with correlation of the pair. In the NL case, a discounting of the consequence of real-world entropy has been justified by assignment of von Neumann’s zero entropy to the “pure” state of superposition. This is formally correct but represents a different issue. It is also irrelevant. From the second law perspective, what matters is the change in state of the ‘entangled’ system between the transition generating it and the measurement. The difference in entropy between populations evolving from generation to measurement, either in superposition or as discrete entities is, for thermodynamic purposes, inconsequential because the starting and ending states are the same. Under either LR or QM predicates, the properties of the pair are correlated because they are generated in a common transition. It is immaterial whether the post-transitional state separates to discrete species immediately, only on measurement, or at some time in between; correlations are conserved, and two separate entities with real properties are identified on measurement. But this does not ameliorate the entropic problems under (a) above.

There is no escaping the fact that if full-visibility is measured starting with a stochastic source, an ordering of the initial state must have occurred. Even for protocols starting with a well-oriented population, a work term to account for ordering associated with rotation of the frame would also be needed to explain full-visibility at all polarizer settings. Any hypothesis to explain this behavior that fails to identify an appropriate work term has, in effect, abandoned the second law, and demands critical scrutiny.

Section 3. Experimental outcomes

i) The Freedman and Clauser analysis, and subtraction of “accidentals”

To satisfy conservation laws, photons of a pair generated in each cascade propagate in opposite directions along a selected axis ($\pm z$) to two stations. The partners have different colors (at 5513 (g) and 4227 Å (b) for the Ca-cascade in $^6$), determined by the energies of the decay steps from the excited state. To maximize coincidences, photons were selected at the two stations on the $\pm z$ axis, and by different color filters, and measurements were constrained to a narrow time window (6-15 ns), so that a coincidence could be measured only if photons had propagated in opposite directions, their colors differed, and they arrived at the “same” time. Because of technical limitations in the early experiments $^{34}$, yields had to be corrected using the Freedman and Clauser accounting $^{1,2,6}$, and adjusted for “accidentals”. The accounting was framed within NL predicates, and the results were claimed to
meet the expectations, and have therefore generally been taken as demonstrating the non-local picture.

As noted above, Fry and Thompson\textsuperscript{13} (with a \textsuperscript{200}Hg-cascade) were the first to use laser excitation, which improved intensities, but introduced a complication. The atom beam was pre-excited to the \(6^3P_2\) state by electron bombardment, then excite at 5461 Å to the \(7^3S_1\) state by a laser polarized parallel to the \(z\)-axis (of propagation), which decayed in a cascade to generate photons at 4358 Å and 2537 Å. The polarization of the laser photo-selected from within the stochastic population, a \(\cos^2\theta\) distribution of transitions with vectorial contributions in that orientation. The populations measured in the plane orthogonal to the \(z\)-axis would then have been isotropic in the \(x, y\) plane, and this was apparent in the mean yields of the singles-counts at the two stations, which were independent of polarizer orientation. The full-visibility behavior implicit in the orthonormal treatment requires a population of pairs with \(H\) and \(V\) photons in dichotomic distribution. The atoms photo-selected along the \(\pm z\)-axis, but with a vector tilted from the \(z\)-axis would emit photons with vectors tilted in the orthogonal plane, and these would therefore have been seen as isotropic by the polarizer in that plane orthogonal to \(z\). However, when measured along the \(y\)-axis, orthogonal to \(z\), the population was polarized\textsuperscript{13}. This might seem as expected, but one feature has not received attention; there was no indication of disorder introduced by the dichotomic state (required in the NL treatment).

The studies by Aspect et al.\textsuperscript{35-37} using a Ca-cascade, improved both photon flux and the efficiency of measurement. The excited state was populated through two-photon absorption, using separate lasers with parallel orientation of their polarization. Orientation was likely along the \(z\)-axis, to generated a population of pairs similar to that in\textsuperscript{13}, so that the wide aperture optics (64°) would sample a similar isotropic distribution in the \(x, y\) plane, giving the same rotationally invariant local means. The coincidence counts (analyzed using the Freedman-Clauser approach) showed a rotationally invariant sinusoidal outcome, as expected from NL premises. To compensate for uncertainties in timing (the life-time of the intermediate states (~5 ns), and the time-spectrum of coincidences (17 ns wide)), coincidences were selected in a time-window of ~20 ns\textsuperscript{35}, and corrected by subtraction of “accidentals” determined from the coincidence rate in the same width of window, but outside the coincidence range.

In later experiments communication loopholes were tested\textsuperscript{36} by rotating the polarizer frame with photons already in flight. The beam in either path could be switched at ~50 MHz to pass (at either station) through one polarizer or another before measurement. Switching did not change the measured amplitude, - an important result because, with the geometry of the setup and the switching
time, this precluded subluminal information transfer. However, the rotational invariance inferred from this behavior would be expected under OL constraints (as in the LR2 outcome); the claim that NL expectations were validated depended on the sinusoidal outcome, on ambiguous correction for accidentals to get full-amplitude\(^3\), and on the closing of the communication loophole, since that was believed (erroneously) to exclude OL interpretations.

In the Freedman and Clauser\(^6\) treatment, the normalized counts, \(R(\sigma)/R_0\), were plotted as a function of \(\sigma\) to give outcome curves, with expectations given by:

\[
\frac{R(\sigma)}{R_0} = 0.25(\epsilon_M^1 + \epsilon_m^1)(\epsilon_M^2 + \epsilon_m^2) + 0.25(\epsilon_M^1 - \epsilon_m^1)(\epsilon_M^2 - \epsilon_m^2)F\theta \cos2\sigma \quad (eq. 6).
\]

Here \(\epsilon_M^1, \epsilon_m^1\) etc., are parallel and perpendicular transmittance values for polarizers at stations 1 and 2, and \(F\theta\) is a function of the half-angle subtended by the light-gathering optics (with value 0.99 at \(\theta = 30^\circ\)). A classical behavior at the polarizers was assumed, as defined by the \(\cos2\sigma\) in the right-most term of \(eq. 6\). The counts (0.1 to 0.3/s) in a coincidence window of 8.1 ns were accumulate in 100 s spans by cycling the system in different states of the polarizers for \(\sim200\) hours, and the count was then corrected for “accidentals” (\(\sim3\%\) of the counts) measured in a separate coincidence channel displaced in time by 50 ns. It was assumed that after correction, the remaining counts represented the outcome of \(eq. 6\).

In the Aspect experiments, coincidence counts were accumulated in a window (\(\sim20\) ns), the duration of which was based on the action profile (the time-spectrum). This was obtained when the signal from one station was delayed with respect to that from the other and varied so as to “sweep” through the point of equal delay. At that point, the profile showed a sharp rise (\(\sim6\) ns) to a maximum, taken to reflect the onset of coincidences. A slower decline (\(\sim90\%\) complete at 16 ns) was taken to represent the scatter of coincidences. The 20 ns window of accumulation therefore collected most coincidences, but also accumulated a substantial pseudo-count from “accidentals”, corrected as above.

\(a)\) Problems in the treatment. There are several obvious problems with \(eq. 6\). As noted in the main text, the first term on the right (RHT1) is not dependent on \(\sigma\). It is the product of two terms used for normalization, \(R_1/R_0 = 0.5(\epsilon_M^1 + \epsilon_m^1)\), and \(R_2/R_0 = 0.5(\epsilon_M^2 + \epsilon_m^2)\), where \(R_1\) is the coincidence rate with polarizer 2 removed, \(R_2\) is the coincidence rate with polarizer 1 removed, and \(R_0\) is the coincidence rate with both polarizers removed. Both terms give a value \(\sim0.5\), independent of polarizer setting. Consequently, the cross product (\(\sim0.25\)) from these terms is empirically
determined and constant, and it could not contribute to the sinusoidal component of the outcome curve. Only the second term on the right (RHT2), with value constrained to the range 0 - 0.25, depends on $\sigma$. It would provide values to define the sinusoidal component, but it could account for only half the full-scale outcome claimed.

There is a second, more subtle problem; the two RHS terms represent different processes. RHT1 uses the same algorithm as that used to score “accidental” counts. In eq. 6, the counts are taken in the coincidence window; the “accidentals” are counted over the same width but outside the coincidence window (so these counts are unrelated to the pairwise coincidences) to provide an estimate of the counts that would have occurred during the coincidence window. The equal weight given to the two terms in eq. 6 is then artificial; the “accidentals” measured are proportional to the window width. In a well-designed protocol, the coincidence window can be reduced to the life-time of the generating process, minimizing the “accidentals”; this ideal can be approximated in well-designed PDC-based protocols. In the ‘idealized’ simulation, where the coincidence window is vanishingly narrow, the “accidentals” do not need to be considered. In different experiments reported, the width of the coincidence window used has varied widely, and the fractional contribution of “accidentals” has varied accordingly, approaching the ideal more closely in\textsuperscript{6} than in\textsuperscript{35-37}. In practice, the “accidentals” measured outside the coincidence range were subtracted from the counts measured in, and the curve reported then had a minimum at zero. In effect, this would eliminate the counts from RHT1.

b) Caroline Thompson’s analysis. It was assumed in the early work that the corrected outcome would represent the overall yield of true coincidences scaled to 0.5 (the sum of the two RHS terms of eq. 6). In analysis of the treatment of “accidentals” from Aspect’s thesis (the only set of data available to her) Thompson\textsuperscript{33} had noted that the experimental curves reported in all papers following the earlier Freedman-Clauser treatment were displayed in a scale with a minimum at zero, with amplitude close to full scale of 0.5 (i.e. without offset). Her own analysis through integration of the Malus’ law outcome for an isotropic population of pairs yielded the same result as my LR2 outcome (half-visibility curves offset from zero). The reported curves could be explained only if the correction for “accidentals” compensated for the offset (as above), and the curves were rescaled. She suggested an alternative treatment for accidentals that yielded from Aspect’s data her analytical outcome (equivalent to my LR2). The analysis of the LR2 outcome through the green curve envelope (Fig. 6 in the text) gives essentially this same result.
For consideration on an orthonormal basis, a wavefunction carrying dichotomic spin properties is needed, for example, \( |\psi\rangle = \frac{(|H_{A}H_{B}\rangle + |V_{A}V_{B}\rangle)}{\sqrt{2}} \), appropriate to a cascade. In the standard treatment, the matrix operations would align frames, and on measurement, the properties actualized would give outcomes following Bell’s QM expectations. This outcome would need only the right-most term of eq. 6, giving

\[
R(\sigma) = 0.5(\epsilon_{M}^{1} - \epsilon_{m}^{1})(\epsilon_{M}^{2} - \epsilon_{m}^{2})F_{\theta}cos2\sigma \quad (eq. 7)
\]

Indeed, in Clauser and Horne\(^1\), the right-most term is the only one included in analysis.

In the cascade results, the rotational invariance of the singles-counts is not controversial. It shows that the population was not polarized in the plane orthogonal to the axis of propagation. As noted in the text (Part A, 5 ii)), it is well explained by a \(\psi\)OL model without invoking dichotomic spin states. For pairs at stochastic orientation, the rotational invariance of the coincidence counts (the LR2 curve) is also uncontroversial. However, full-visibility of the NL expectation would be seen only if, on actualization, each pair from the cascade had become aligned with the polarizer frame. If partners had the same orientation, and were not dichotomic, that alignment would lead to polarization in the plane. Only a dichotomic partitioning of orthogonal pairs (nominal in \(HH\) or \(VV\) orientation) could explain the invariance of the singles-counts. The matrix operation giving alignment for NL expectations also requires an ontic dichotomy. However, as noted in the main text (Part B, 6 2)), there is no justification for any assumption of ontic dichotomy in a stochastic population of photons.

\(\quad ii) \quad \text{Experiments using PDC sources}
\)

\(a) \quad \text{Early Experiments.} \) Protocols for entanglement experiments using PDC sources were established in papers starting in the early 1990s. In the seminal paper by Kwiat et al. \(38\), the PDC output was initially “prepared” by selecting beams from the mixed populations at the two cone intercepts, and sending the two beams to the separate stations. Before the two beams diverged, HWP0 was set at \(45^\circ\) across both to switch \(HV\) to \(VH\) and vice versa in the mixed populations. The rationale was to make it possible to eliminate the time lag (~385 fs) arising from different refractive paths in the crystal (this difference would in principle allow \(H\) and \(V\) photons to be distinguished), which could be compensated by insertion of additional BBO crystals at correcting orientations in the beams, making them indistinguishable. This would not modify the classical expectations. Further preparation by insertion of additional half- (or quarter-) wave plates in one or other beam could be used to implement different Bell states (although tests using the intrinsic \(HV/VH\) state did not need them). However, all
rotations required in analysis were accomplished using additional HWPs (P1 and P2) to rotate the beam in front of fixed polarizers. A footnote suggests that “It is necessary to examine the case with one of the polarizers at ±45° in order to demonstrate the quantum coherence between the terms in the entangled states…” This note invokes the canonical settings for \( \alpha \) and \( \alpha' \) used to distinguish Bell’s zigzag from the NL sinusoid, but this is not a necessary configuration; any departure from the aligned condition could be used to test for the special rotational invariance and could be more simply implemented. Any such test would require clear specification of all rotational frames. Unfortunately, in the discussion it is often unclear, either what reference frame was used, or what relative orientation of frames was achieved. A figure legend states that polarizer 1 was fixed at \( \theta_1 = -45^\circ \) and used as reference in determining angle difference from \( \theta_2 \) (for the other polarizer), but elsewhere in the text, \( \theta_1 \) and \( \theta_2 \) are settings, now associated with HWPs P1 and P2. The settings for P1 and P2 were changed in increments of 22.5° typical of canonical settings (±\( n \frac{\pi}{8} \) with \( n = 1 \) or 3 for \( \theta_1 \) rotations, and \( n = 0 \) or 2 for \( \theta_2 \) rotations). However, the most straightforward reading would be that the \( \theta_1 \) and \( \theta_2 \) rotations reflect angles at which P1, P2 were set. With polarizer 1 at ±45° or polarizer 2 at 0°, and with either setting chosen as the reference frame, the P1 or P2 rotations, respectively, would then maintain alignment of photon and polarizer frames, and full visibility on rotation of the other (variable) beam would be vOL compliant (Fig. 8A of main text). Alternatively, if polarizer 1 was fixed at 0°, rotation of P1 by ±22.5 or ±67.5 would set the beam at 45° to the polarizer so that any photon would have an equal probability of emerging in the \( H \) or \( V \) beam. This would scramble the assignment of coincidences and eliminate visibility. If HWPs simply rotate the beam, observation of full visibility at these settings would be inexplicable under any model invoking Malus law. (In simulation, rotation of the beam by these angles generates a curve of reduced amplitude but little interest.) Because only one detector was use at each station, P1 and P2 were also used to implement rotations needed to partition photons to ordinary or extraordinary rays (the polarization analyzer function) to further complicated analysis. The ambiguities preclude definitive simulation, or any firm conclusion as to which model best explains this data set.

In Fiorentino et al. a similar initial state preparation was used for time-tag corrections; Kim et al. used a Sagnac interferometer configuration, also generating \( VH/HV \) populations, to achieve a similar effect. In both, HWPs were used for the discriminator function, by rotation the beam in front of fixed polarizers at the different stations (this part of the general protocol was similar in the two reports). In both papers, the angles reported reflect the angle by which the beams were rotated (\( \theta_A \) or \( \theta_B \) are 2 x the angle to which the relevant HWP was set). The text implies that both polarizers were fixed
at the same orientation (0°). For the outcome curves shown, the variable analyzer was at station A, and that at station B was fixed. Coincidence counts were plotted versus angle difference, defined as “…analyzer angle $\theta_A$ in arm A for analyzer angle $\theta_B$ in arm B set to…” either 0°, 45°, 90°, 135° or -45° (here, the term “…set to…” has the meaning “…set to implement a rotation of…”).

In Fiorentino et al.39, the pump laser excited PDC in a periodically poled KTiOPO$_4$ crystal to generate collinear $H$ and $V$ output beams, with entangled partners in opposite hemispheres. The superposition then had wavefunction $|\psi\rangle = \frac{(|H_AV_B\rangle + |V_AH_B\rangle)}{\sqrt{2}}$. The top and bottom hemispheres were separated by irises so that both had an equal mix of uncorrelated $H$ and $V$ photons, and the partners in top and bottom were separated by irises before one set or the other was directed to each station.

**b) Polarization Sagnac Interferometer configuration**40. This arrangement involves two distinct PDC process. The pump laser, presumably after rotation of the beam by $\approx 45°$ at the HWP1, was directed via a dichroic mirror onto the PBS which partitioned the pump beam into separate polarized components, $E_H$ and $E_V$, exiting the PBS in orthogonal beams. The separate beams provided inputs into the Sagnac interferometer, directed by mirrors to follow counter-clockwise (ccw) ($E_H$) and clockwise (cc) ($E_V$) paths through the interferometer. Both beams traverse the same PDC crystal, a KTiOPO$_4$ crystal in the diagonal section for PDC, but from opposite directions. In both clockwise and counterclockwise paths, PDC converted the pump beam into two collinear output beams with $H$ or $V$ components, labeled signal and idler. These were passed back through the same mirrors to the PBS. The path was supplemented by an essential additional component, a HWP at 45°, inserted in the vertical arm. This HWP2 (which rotated the beam by 90°) precedes the PDC process in the clockwise ($E_V$) channel, but it intercepts the PDC output in the counter clockwise channel. The rotation by 90° in the $E_V$ path ensures that in both paths, the pump beam arriving at the crystal was in the same $H$ orientation. As a consequence, in both PDC processes signal and idler outputs were initially in the same phase ($H_SV_I$). In the $E_H$ path, the HWP flipped the $|H_S\rangle|V_I\rangle$ kets from the counterclockwise beam after PDC to $|V_S\rangle|H_I\rangle$. Recombining the beams at the PBS then mixed the outputs to give wavefunction, $|\psi\rangle = \frac{|H_S\rangle|V_I\rangle_2 + |V_S\rangle_1|H_I\rangle_2}{\sqrt{2}}$, with two populations, labelled signal and idler, directed to separate measurement stations. Station 1, the signal channel, then received $H_S$(cw) and $V_S$(ccw) photons, and station 2, the idler channel, their “entangled” partners, $V_I$(cw) and $H_I$(ccw) photons.

After encountering the PBS on exit, an equal mix of $H$ and $V$ photons go to each station, as shown in the wavefunction equation. The correlated pairs are $H_S$(cw) with $V_I$(cw), and $V_S$(ccw) with $H_I$(ccw); beams $H_S$ and $V_S$ come from separate PDC processes, and likewise, $H_I$ and $V_I$. Since the $H$ and $V$ photons going to a measurement station come from different, uncorrelated, PDC processes, at
neither station is the population in the ontic dichotomy required for the NL treatment; the polarized beams in this mix are not the same as the dichotomic mixture from the correlated beams generated from the intercept of entangled cones from PDC in earlier protocols. At the measurement stations, correlated partners are picked out of the mix by temporal coincidence.

c) Simulation of the Sagnac PDC processes. The fully polarization state of each output beam could not have been detected experimentally because in the Sagnac configuration, an “interference” between opposite phases of the PDC output beams at the PBS on exit leads to the mean yield of 0.5 at each detector, and the populations would appear to be dichotomic. However, the program can be used to simulate each process, and demonstrates that all beams arriving at a measurement station must be polarized (Fig. SI_1A). The separate processes can be modelled using the PDC-EO option, in which the $H_S$ (signal, or Ordinary) and $V_I$ (idler, or Extraordinary) rays from PDC are separated and passed to different stations. In Fig. SI_1B (simulating the $E_V$ (clockwise) channel), the left panel shows the coincidence count when the PDC output is in the same frame as the fixed polarization analyzer (with Ordinary channel collecting $H$ photons, Extraordinary $V$ photons). Despite the polarization, this shows the full-visibility of the coincidence count expected from NL or νOL models. The right-hand panel shows the singles counts at detectors Q (red, O1), R (blue, E1), S (yellow, O2), and T (green, E2), with symbols colored as indicated. These show the polarization; with the $H$ photons directed to station 1, all photons were detected in the ordinary ray (O1) and none in the extraordinary ray (E1); at station 2, receiving the $V$ photons, where polarizer 2 was varied, complementary sinusoids were detected. In Fig. SI_1C (simulating the $E_H$ channel), a HWP at 45° was inserted in the PDC output so that the $H_SV_I$ output was rotated to $V_SH_I$, with the $V$ photons going to station 1. The coincidence counts shown in the left panel showed the same full-visibility coincidence count, but the pattern of singles counts at the detectors was reversed, as expected. It will be obvious that summing the outcomes at each detector (Fig. SI_1D) would give the result seen experimentally (the mean of 0.5 at each detector, 2 photons overall). However, it is also clear that the “interference” seen in the singles counts is not a wavy phenomenon, but a simple subtraction.

d) Anomalies in the behavior at the HWP in channel B. Despite the complications above, the numerous refractive manipulations contributing to state preparation were well-defined, and the results appear to be definitive. The output from the Sagnac interferometer consists of the four polarized beams O1, E1, O2, E2, distributed to measuring stations as above, each equipped with a polarization analyzer consisting of a HWP/polarizer combination. Since HWPs were used to provide the discrim-
inator function, the polarizer in the variable path must have been set at 0\(^\circ\). Since \(\theta_A\) and \(\theta_B\) are presented as comparable, and no other information is provided, this might imply that both polarizers are set to 0\(^\circ\). Is that what the data show? The coincidence curves show full-visibility at all angles to which analyzer \(B\) was set, including ±45\(^\circ\). This result is contrary to the zero-visibility demonstrated at settings of ±45\(^\circ\) in my vOL simulation, and also expected experimentally if the analyzing HWP implemented the simple beam rotation implicit in the protocol design. Two features of this outcome deserve further comment.

(i) Given that discrimination is implemented at the HWPs, the analytical steps must be modified from that in Shimony’s NL treatment, where actualization occurs with respect to the polarizer itself. Actualization must occur earlier, before the discriminating HWPs, and both partners would have to be actualized in an orientation appropriate to the function of the HWPs (the rotation of the beams posited). The angles would also have to be appropriate to the polarizer frame and the outcome would be either \(H\) or \(V\), as shown in the descriptive scheme\(^{40}\). However, those angles are determined by a well-defined set of refractive and reflective events, meticulously catalogued in the protocol, starting with the pump laser, and culminating at the discriminating HWP as shown on the schematics; there is nothing indeterminate about them. There is therefore no need to postulate their generation by actualization from a superposition. That seems to make the whole NL treatment redundant.

(ii) Experimentally, the outcome curves were shown to be shifted in phase with respect to the frame provided by the polarizer in channel \(A\) fixed at 0\(^\circ\). The shift was by the angle to which the HWP analyzer in path \(B\) was set; for example, with \(\theta_B\) at 0\(^\circ\), coincidence count was minimal when \(\theta_A\) was at 0\(^\circ\), but with \(\theta_B\) set at 45\(^\circ\), the count was minimal when \(\theta_A\) was set at 45\(^\circ\). The frame shift demonstrates that the population of pairs were rotated as claimed, but there are a couple of odd features. The frame shift shows that the rotation in path \(B\) was independent of the fixed polarizer in path \(A\). No simultaneous actualization referring both photon frames occurred in both channels as expected in the conventional NL hypothesis. To explain the behavior by special changes or events associated with the NL process would require a whole series of \textit{ad hoc} assumptions (see above) and can therefore be discounted as a serious possibility.

Secondly, the visibility of the curve determined experimentally was the same at all settings for \(\theta_B\). If polarizer \(B\) was set at 0\(^\circ\), and had shown Malus’ law behavior in analysis of the photons, and the HWP implemented a simple rotation, the full visibility (seen with \(\theta_B\) at 0\(^\circ\) or 90\(^\circ\)) would have been entirely lost with \(\theta_B\) at 45\(^\circ\) or 135\(^\circ\). After rotation to ±45\(^\circ\), any photon would have an equal probability of distribution to ordinary and extraordinary rays of the analyzer, and their nominal spin
would be indistinguishable in comparisons with their partners. The reported behavior would therefore be incompatible with the Malus’ law behavior of polarizers. The behavior can only be accounted for if some interplay occurred between the analytical HWP and the polarizer functions so as to preserve the nominal spin property in the output rays of the analyzer.

One obvious possibility lies in an ambiguity in the description “…analyzer angle $\theta_B$ in arm $B$ set to…” either 0°, 45°, 90°, 135° or -45°…”. This could legitimately be interpreted as meaning that $\theta_B$ angle specifies both that by which of the beam was rotated by the HWP, and the setting of the polarizer in arm $B$. As shown in Fig. SI-2A, setting both values the same generates the outcome claimed without the need for anything more complicated (this is also shown in Fig. 8A of the main text, as an explanation for the results in38). The same protocol can be applied in simulation of the conventional $VH/HV$ PDC output (cf.39), which, as noted above, gives a similar behavior on measurement of coincidences (Fig. SI-2B). Since both these simulated outcomes are perfectly compatible with locality constraints, and the protocols match those reported, and since the manipulations in the Sagnac interferometer (or in generation of the conventional cone overlaps), are meticulously detailed and produce the same population at the measuring stations with or without “quantum magic”39,40 (see (i) above), a local realistic explanation is perfectly sufficient to account for this behavior.

A second possible explanation in either of these experimental setups, would be a different protocol that can, under vOL constraints and with the fixed polarizers at 0°, readily demonstrated outcome curves essentially the same as those reported. When the analyzer at station $A$ is used as the fixed reference, and that at station $B$ as the variable analyzer implementing both the offset rotation and analytic rotations, the outcome is that demonstrated in41, and similar to that in39,40 (see below). This configuration is simulated in the program, choosing station 1 as $A$ (fixed at 0°), and 2 as $B$, and by setting HWP2 at $\theta_B/2$ and varying the setting of polarizer 2 to scan through angle-differences (Fig. 8B of the main text).

e) The polarization Sagnac interferometer configuration has been used in other recent reports. It is the source of “entangled” photons in the satellite-based experiments of Yin et al.18. All the beams from PDC projected to earthbound stations had signal and idler components polarized. The large separations between source and detectors, variation in orientation at the satellite, and variation in distances between source and stations, demanded an explicit implementation of spatial and temporal coordination between the three frames. This was achieved by projection of additional laser beams of different color, one pulsed to allow (in coordination with synchronized atomic clocks at all
stations) for precise measurement of distance, and the other polarized so as to allow alignment of frames. There was no suggestion that these supplementary beams were other than determinate in Einstein’s sense. With these additional refinements, the outcome reported was essentially the same as demonstrated in Kim et al.\textsuperscript{40}, simulated in Fig. SI-2A. More recently, the Sagnac configuration has been used as the source in Handsteiner et al.\textsuperscript{42}, where switching of the beams arriving at spatial distant measurement stations by EOMs was based on the color (blue or red, selected by a beam splitter at 700 nm) of light from distant galaxies, with a different cosmic source chosen at the two stations. The switching of the beams, in each case by 90°, “…induced a measurement in the following linear polarization bases for Alice: 45°/135° (blue) and 0°/90° (red), and Bob: 22.5°/112.5° (blue) and −22.5°/67.5° (red), respectively…”. No other information about the discriminator function is given, but if the “polarization bases” were implemented with HWP and polarizer correlated as above, the result would have been \textit{vOL} compliant. From the different combinations, canonical $S$ values in the range 2.4-2.5 were determined, substantially exceeding the OL limit of $\leq 2$, but this alone is uninteresting.

\textbf{f) In the reports of Fiorentino et al.\textsuperscript{39} and of Kim et al\textsuperscript{40} discussed above, it was implicit in the comparison between stations that both polarizers were fixed at 0°. On rotation of the beam by 45° before falling on a fixed polarizer set at 0°, the photons under Malus’ law expectations could then not be distinguished as $H$ or $V$. Experimental results showing full-visibility rotational invariance would then not be possible. If refractive engagements are quantized and local, then, as discussed above, actualization would have to occur at the first refractive component encountered. However, in the protocols reported, that would restrict superposition and the associated quantum “magic” to a few picoseconds before state preparation, making both in effect redundant. The results could then not be explained by the NL model because entities are actualized before state preparation, and would then follow the same classical constraints as my \textit{vOL} photons. The results claimed would then be in contradiction with the empirical behavior of polarizers in either perspective. To get over this problem, any explanation for full-visibility would require an interaction between the beam and the polarizer in which the distinction between $H$ and $V$ orientations was conserved in the polarizer output. I showed above that a local realistic explanation consistent with the published protocol was straightforward, - the HWP and polarizer were rotated together. If that interpretation proves wrong, an alternative explanation for the conservation of spin information is needed. The possibilities are limited. HWPs differentially retard linearly polarized light in $H$ and $V$ orientation. When the HWP is rotated away from the reference frame, the photons (depending on angle difference), are partitioned so as to follow $H$
and $V$ paths of different refractive index, and so are differentially retarded, introducing a phase delay between them. This would leave them, in principle, still distinguishable at the elemental level. The $H$ and $V$ photons at the two stations could be distinguished at the polarizers based not on their orientation but on the phase difference, elliptical tweaking, etc. A recent report from a dominant group has suggested a possible explanation incorporating such thinking: “...a polarization qubit being in a coherent superposition of horizontal and vertical polarizations (with a certain phase relation) can be understood as a photon polarized diagonally at $\pm45^\circ$. A polarizer set at this angle will always transmit such a photon with 100% probability (and zero probability when set to $-45^\circ$)…” This phrase is somewhat incoherent; the superposition is clearly essential, but is used to specify the potential configurations of a single entity. If needed for the weird behavior at the polarizer, the entangled state could not yet have been actualized. Can such a description be of use in analyzing an entangled pair, where the key element is the correlation between them? This begs all the questions inherent in the problems discussed above. Where does actualization occur? At the polarizer? The HWP? Why there rather than at the first refractive component? It seems that the polarizer is set at $45^\circ$, but if the $H/V$ state for one photon can be understood as “…polarized diagonally at $45^\circ$…” is the other at $-45^\circ$? Unless these are actualized entities, either possibility would have to be included in the superposition, in which case, on examining a population, wouldn’t the polarizer reveal the same uncertainty as in the classical case? On the other hand, if the “polarization qubit” at $45^\circ$ is a single entity in a population, and has a partner at $-45^\circ$ in a correlated population, and “coherent superposition” simply acknowledges that pairs are correlated, then if the HWP rotates the $H/V$ state by $45^\circ$, and the polarizer is set at $45^\circ$ (the condition simulated in Fig. SI-2A), the $45^\circ$ and $-45^\circ$ photons are distinguishable, and the outcome would be fully consistent with local realism.

g) Other reports of pertinent experimental outcomes. The behavior simulated in the second possible explanation above has previously been demonstrated by Weihs et al. Communication loopholes were explored using an electro-optic modulator (EOM) to switch one beam between 0° and 45°, and their data showed full visibility on switching, interpreted as showing the full-visibility rotational invariance. All results reported were from experiments in which the polarizer at one station (Bob’s) was fixed at 0°. Both variation of angle to generate the outcome curve, and rapid switching of the beam (by 45°), were then implemented using the EOM at the other station (Alice’s). The change by 45° shifted the phase of the full-visibility sinusoid by the switched angle (the curves in Fig. 8B of the main text also simulates this case). The phase displacement at Alice’s station and the full-visibility
would be compatible either with NL predicates or with vOL constraints, so could not distinguish between models. However, closure of the communications loophole strongly favors the vOL interpretation (see Main text, Communication loopholes).

In all these reports using PDC sources, results claimed to exclude local realism can be reproduced in vOL simulation either directly, or with trivial changes in protocol, using interpretations consistent with the experimental protocol as described.

\textbf{h) Partial entanglement.} As an example of a more recent paper, in a “detection-loophole-free test of quantum nonlocality” by Christensen et al.\textsuperscript{45}, a sophisticated apparatus was built to take advantage of an analysis by Eberhard\textsuperscript{46} showing that with non-maximally entangled states (with $|\psi_r\rangle = (r|HH\rangle + |VV\rangle)/\sqrt{1 + r^2}$), the detector efficiency required to allow discrimination between the standard BCHSH models could be reduced, bringing such discrimination within the range of the relatively high-efficiency detectors then available (75\% ±2\% after accounting for losses). The purpose of this exercise can be discussed with reference to a rearranged version of the Clauser-Horne\textsuperscript{1} inequality,

$$B' = \frac{p_{12}(a, b) + p_{12}(a, b') + p_{12}(a', b) - p_{12}(a', b')}{p_1(a) + p_2(b)} \leq 1.$$ 

By using a small value of $r$, one can choose discriminator settings $a$ and $b$ to nearly block the vertically polarized singles counts (the denominator), while choosing $a'$ and $b'$ to maximize the numerator. Discriminator settings, coincidence counts, singles counts, and numbers of photons detected at each station for the different settings were determined, and used to calculate a normalized set of outcome values. The value for $r$ was varied to find, at $r = 0.26$, a degree of entanglement that maximized $B'$ at $1.015 \pm 0.002$. The value of $B' > 1$ demonstrated was equivalent to exceeding the limit of $\leq 2$ with high significance, and the main conclusion of the study was that this excluded local realism. Unfortunately, as noted in\textsuperscript{47}, the limit of $\leq 2$ is not of interest in this regard, and the study therefore cannot, on that basis, be taken as excluding local realism. There is still considerable ambiguity in this report in the description of relative frames of the pump source (and therefore for the photons), discriminators, and the angles of the fixed polarizers (not specified). Although in principle my simulation could be extended to deal with the non-ideal entanglement, without clarification of settings, it would not be possible to test whether the rotational invariance claimed was contrary to a simple Malus’ law (vOL) interpretation.

\textit{iii) Recent treatments that have claimed to demonstrate the QM outcome from local realistic models.}
It was claimed in the main text that no local realistic model can predict the invariant outcome expected from QM predicates. Two recent claims have contradicted that conclusion.

De Raedt\textsuperscript{48} demonstrated in a similar simulation, a rotationally invariant full-visibility outcome starting with a stochastic population of correlated LR photon pairs. It is a natural feature of a stochastic population that, for any particular polarizer setting, some pairs would be close to the aligned condition (see envelope in Fig. SI\_1), and selection of only these pairs would generate an outcome approaching full visibility. In de Raedt’s simulation, such a selection (of <1\% of the coincidences, all close to the QM alignment), was implemented through differential time of flight through refractive paths. No entanglement experiment has tested this approach, because the time scale, likely in the <1 ps range (cf.\textsuperscript{38}), is out of the range of protocols reported. However, the de Raedt range is accessible using some of the approaches developed for femtosecond spectroscopy, so this could in principle be tested. Published experiments have evaluated all coincidences measured in a window \textasciitilde 0.7 - 10 ns, so full-visibility results from these cannot be attributed to a de Raedt mechanism.

In a different contradiction, Christian\textsuperscript{49,50} has suggested a disproof of Bell’s theorem based on use of bivectors in the Clifford algebra $Cl_{3,0}$ to represent the discrete vectorial properties of correlated quantum entities. This usage has been vehemently criticized, but the hostility seems to have been more a response to the temerity of his claims than to his use of algebra, since others have used Clifford $Cl_{3,0}$ without complaint\textsuperscript{51,52}. His claims are: (i) that such representation is local and realistic in terms of Bell’s criteria, and more appropriate to vectorial analysis; (ii) that the algebra generates all the outcomes of the orthodox QM treatment; and (iii) that therefore the standard contention that no LR state can do this is disproved. My treatment is similar to his in that both explicitly represent discrete entities, so the outcome of my simulation validates his use of the Clifford algebra, though it shows that the sophistication is unnecessary. Both approaches show independently that superposition is not necessary. His wider claim, that the approach through bivectors can match all the claims of the orthodox treatment, would be defensible if its predicates were valid. However, Christian’s treatment invokes the binary representation of orthogonal correlation (and hence the exclusion of real vectors), and operation of the Pauli matrices for alignment of photon and polarizer frames. If these are valid, he has made his point. On the other hand, the simple and explicit representation of real orientations in my simulation avoids these \textit{ad hoc} features, as discussed in the text; if a realistic treatment requires real orientations,
the special invariant behavior is not possible for an OL state. On the other hand, if the ad hoc assumptions are right for the orthodox QM approach, then they are right for Christian’s approach, and his claim to have disproved Bell’s theorem would have the same validity as the orthodox treatment.

Section 4. Particle/wave duality: where to go from here

It is surprising that so little effort has been made to exploit the informational richness of the PDC output to test the arguments in the entanglement debate. Why not take advantage of the polychromatic source to select phase-matched pairs with partners of different color in the $H$ and $V$ cones (cf. 23,24), so that color-labelling could be used to track them during state preparation? Phase-matched populations of different color necessarily come in cones of different diameter$^{24}$, but it should be possible to design protocols using digital light projection (DLP) technologies and programming of the millions of switchable mirrors available (8.8 M in high-end chips), to select discrete populations of correlated pairs from the parametrically complementary spectra generated in the two cones. This could readily be implemented to provide an Enigma-style machine based on photonic encoding, exploiting the informational richness provided by the polychromatic PDC.

Particle/wave duality allows properties of quantum entities to be interpreted using a different formalism depending on the experimental context. However, quantum entities do not change their nature to suit the thinking; the interpretation is changed to suit both context and theoretical prejudice. In the conventional NL treatment, the wavy realm is implemented through superposition and the matrix operations. In recent treatments, in order to accommodate the “wavy” nature of light (revealed through interference effects like the two-slit experiments with photons, refractive behavior, and similar experiments with electrons, bucky-balls, etc.), Maxwell’s treatment in which light waves interacting through electromagnetic properties with harmonic oscillators has been conscripted to interpretation of the interactions of photons with electrons; Dirac grafted Maxwell’s classical wave treatment onto a quantized treatment of electrons to give us quantum electrodynamics (QED). In extending this, Feynman’s recognized that at the process level, the treatment must be particulate and local. Through his path integral method, he adapted Maxwell’s wave theory to a particulate context through the brilliant sleight of hand of quantizing the paths. He could calculate phase delays between paths via interaction of the electromagnetic
wave with harmonic oscillators, and, from these, the interference effects that underlie the preference for particular set of paths. As noted in the main text, the problem with this approach is that photons are neutral. The interactions through field effects involve assumptions which, because they are constrained by the measurable properties, give results that conform to classical expectations, but at the elemental level they cannot be natural. A more realistic approach is to accept that photons are neutral and recognizes that their interactions with electrons involve momentum transfer; my simulation is a naïve application of this model. Since the simulations, with some plausible interpretation of ambiguities in description of protocols, can account for results reported, its algorithms are sufficient to reconstruct the natural behavior without resort to complex math. I discussed in the main text several mechanistic scenarios in which the observable behavior could be accounted for under local realistic constraints in terms of vector projections for momentum transfer that are the same as those that come into play in NL interpretations after actualization. In sum, these scenarios account for the outcome of entanglement experiments in OL terms, if allowance is made for ambiguities in description of “state preparation”.

In the following section I extend this discussion to areas outside the entanglement debate.

i) Pin-hole diffraction. The main justification for wavy treatments is their usefulness in dealing with interference effects. However, given the low intensities involved, this leads to invocation of interference of one photon with itself, an outlandish concept if refractive exchanges are quantized. Maybe an alternative perspective would be useful. Consider as an example the diffraction from a pinhole (Fig. SI_3). The concentric rings seen at the detector are (nearly) equally spaced from the zeroth ring, for which the angle from the axis of transmission is determined by \( \sin \theta \sim 1.22 \frac{\lambda}{d} \), where \( \lambda \) is the wavelength and \( d \) is the aperture diameter. The photon density at the aperture is such that photons pass through one at a time. With a small aperture, a substantial fraction of the photons will interact with (“feel”) the material of the circumference (this would be described by the “evanescent wave” behavior in wavy treatments), explore electronic displacements, and engage in a refractive event on finding a matching transition. Since this interaction is locally asymmetrical, the photon returned (without loss of energy) will have a trajectory displaced by \( \theta \). Depending on initial trajectory and depth of penetration of the “evanescent wave”, this might free the photon to complete its journey to the detector (a large fraction judging by the intensity of the first ring). However, the possibility of finding another engagement means that re-
peats of the displacement will occur with diminishing probability. Since the probability of finding a matching transition will be stochastic but uniform, and the duration of each event will match the photon frequency, successive events will be approximately equally spaced in time and angular displacement, with scatter about a mean to give the fuzziness. The probability of engagement for any photon in a population will be isotropic around the diameter, giving the circular pattern observed. Recognizing that the number of successive refractive engagements represent different paths to the detector, this scenario might alternatively be formalized through a path integral approach.

ii) Double-slit interference.
a) Similar behavior in the two-slit context could explain the classical interference effects. In that case, we have additional evidence from experiments using weak-measurement\textsuperscript{55,56}. Kocsis et al.\textsuperscript{55} used a quantum dot source to generate a population of single photons, which was split randomly between two fiber optic pathways, configured to emulate the two-slits, then polarized diagonally. This allowed for a treatment in which the diagonal vector of each photon could be treated in terms of orthogonal components: $|D\rangle = \frac{|H\rangle+|V\rangle}{\sqrt{2}}$. Insertion of a birefringent calcite crystal perturbed a sub-ensemble of the photon population. Since the $H$ and $V$ components follow different paths through the crystal, this would lead, in the classical wave treatment, to a phase delay. Experimentally, with the calcite set at an appropriate angle, the $H$ component followed the natural trajectory, while the $V$ component was displaced. Then, by examining the displacement of the two components (determined on conventional (strong) measurement on a CCD array downstream), it was possible, by moving the calcite crystal, to trace trajectories of sub-populations of photons. In the standard QM interpretation, this information should not be accessible, because of the superposition, but the results show trajectories very like those calculated using the de Broglie/Bohm approach\textsuperscript{57-59}. The authors emphasize that the reconstruction of the trajectories was in no way dependent on a choice of interpretation, but it is difficult to escape the conclusion that two types of refractive behavior were involved: (i) since the trajectories start at the slits and account for the interference pattern, the behavior must reflect photons at the slits engaged as discrete entities in repeating quantized interactions leading to a pattern of paths, essentially as in the pin-hole diffraction above; (ii) the weak measurements involve local interactions of discrete photons with the calcite probe.
in which the refractive behavior maps those paths; \textit{(iii)} these local interactions can account for the interference observed through the classical geometric parameters.

b) The de Broglie/Bohm approach also features in a recent paper treating the two-slit experiment by Aharonov et al., in which they have suggested “...a time-symmetric interpretation of quantum mechanics that does not stem from the wave properties of the particle. Rather, it posits corpuscular properties along with nonlocal properties, all of which are deterministic...”, and “…this change of perspective points to deterministic properties in the Heisenberg picture as primitive instead of the wave function, which remains an ensemble property. This way, within a double-slit experiment, the particle goes only through one of the slits....”. In this treatment, Einstein’s “elements of reality” are clearly recognizable, and his perspective rather than Bohr’s is the dominant one. Although in their further discussion, nonlocal properties were required to account for involvement of both slits, in the light of i) and ii) above, these examples would be quite consistent with real properties of photons and local quantized interactions and would not contravene relativistic constraints.

\textit{iii) Wavy and particulate treatments give the same result.} Photons at the “single-photon” intensities normally used in diffraction experiments arrive at the detector as discrete entities, and likely travel separately too. Refractive behaviors in the “wavy” QM interpretation are treated as involving separate $H$ and $V$ vectorial components of the wave travelling through different refractive paths to generate a phase delay. Could the phase delay be exploited in distinguishing between local and non-local models? Extrapolation of this wavy view to single photon intensities leads to a paradox: could two vectorial components be contributed by the same photon? That would be nonsensical in a quantized context; detection for the $H$ and $V$ components in the “wavy” interpretation cannot represent two events detected from the same photon. The two components must represent two separate photons taking different paths, just as they do in the vOL model, and any conclusion equivalent to the wavy one would depend on measurements on populations. The results in either case must be the same; separate photons are detected, but with the statistical distribution from a population. The integral over many paths in Feynman’s approach would give the same result.

\textit{iii) Momentum borrowing.} Under NL premises, expectation of full-visibility in entanglement experiments is the same no matter the degree of order in the population. A work-term is
then required to drive the necessary pre-ordering. The fact that none is suggested certainly weakens the NL case. Are there possibilities that could salvage it? In the canonical (Minkowski) treatment, the momentum of the photon \( p = E(n/c) \), where \( c \) is the speed of light in vacuo and \( n \) is the refractive index of the medium) is lost from the low, and gained by the high refractive medium in proportion to the time difference (as determined by the path length and \( c/n \)). Overall, the energy of the photon, \( E \), is conserved, and the momentum is returned in full on exit, but only after a delay. However, while engaged, the transferred momentum accelerates the medium (think molecular tweezers, light sails). The “lost” momentum borrowed from time could perhaps provide a work-term, but by what mechanism could this be used to re-order the stochastic or disordered source? Events observed through a window lag events observed through an open path by 20 picoseconds or so. Since the observer through a glass sees events later than an observer at the same distance with no interruption, there is at least in principle an opportunity for the latter to take advantage. However, in the context of a space-like separation of stations, no obvious scenario would provide enough advantage to explain away the problems with relativity and the second law endemic to NL treatments.

Section 5. Momentum transfer on a wider scale

In extension of NL treatments, Dirac’s adoption of Maxwell’s theory has been favored because, in the quest to unify quantum mechanics and relativity, field theories seem better suited to gravitational fields. As succinctly put in a recent article, “…All the fundamental forces of the universe are known to follow the laws of quantum mechanics, save one: gravity. Finding a way to fit gravity into quantum mechanics would bring scientists a giant leap closer to a ‘theory of everything’ that could entirely explain the workings of the cosmos from first principles…”. However, no satisfactory theory of gravity has emerged, and it is therefore worth considering some unconventional scenarios.

In the current consensus, between 10 s and 46 Ky after the big-bang, a substantial fraction (at least a half) of the mass-energy in the universe was in the form of photons, trapped in a dense plasma by frequent interaction with electrons (and initially positrons), protons, and nuclei. Over the first minutes, as the universe expanded and cooled, neutrons were able to combine with positrons to form ionized nuclei (\( \text{H}, \text{H}, \) and by nucleosynthesis, \( ^4\text{He} \)), leaving electrons, and an excess of protons over neutrons. These could still interact with photons, keeping them trapped.
Starting at about 377 Ky, further expansion and cooling allowed combination (deionization) of electrons and ionized nuclei to form neutral atoms and molecules, lowering the probability of collision. Expansion over this time also extended the free-flight time of the photons. As the probability for free flight exceeded that for collision, the photons could decouple from their interactions and escape, and by about 700 Ky, they were all gone, and the universe entered its ‘dark ages’. This batch of photons, now red-shifted, have left their signature in the cosmic microwave background. Light did not appear again until 400 My after the big bang, in the reionization phase, when the first stars began to form.

It will be obvious from this evolution that on decoupling, a substantial fraction of the mass/energy of the universe was released as photons. What has happened to that energy? The conventional wisdom is that the photons have mostly disappeared into the infinity of the void, because, in the present universe, it can be readily calculated from the spectrum of cosmic microwave background (CMB) (an almost perfect black body curve, peaking at ~5.5 cm⁻¹, in equilibrium with the temperature of the universe, ~2.76K) that the mass-equivalent density of radiation is ~1/10000 of the total mass density. However, in the deionization phase, with T ~4000 K shortly after the big bang when light could first escape, this ratio was close to 1. The radiation densities of these “thermal” photons are calculated from equilibration of photons with the universe as a black body; the change in this ratio over that time mainly reflects the change in temperature following some 13.8 billion years of further expansion.

Obviously, this approach through temperature does not address the question above. Firstly, the temperature change of the universe tells us nothing about the fraction of mass-energy lost. Secondly, the CMB represents only the high-grade photons, which after being red-shifted are still in a narrow energy range defined by a near-perfect black body curve. Thirdly, from the preceding, these photons have not been involved in any significantly dissipative interactions, which would have shifted their spectra further to the red.

Dissipation occurs on absorption. The visible photons in the present universe are generated by stars at high temperature and energy. An unknown fraction will engage in dissipative processes in which the energy absorbed is eventually returned as photons but at a lower temperature and frequency; in general, since, overall, energy is conserved, for any dissipative process recycling photons, \( E_{in} = n_{in}h\nu_{in} = E_{out} = n_{out}h\nu_{out} \), and \( n_{out}/n_{in} \) increases in proportion to \( \lambda_{out}/\lambda_{in} \); the dissipation is reflected in an increase in the number of photons at lower energy. A local example
can be found by plugging \( v_{in} \) and \( v_{out} \) (derived from the spectral distribution of sunlight and ‘earthlight’) into the above equation, to show every incoming photon from the sunn hitting the earth creates \( \sim 34 \) going out.

What happens on a cosmic scale? Successive dissipative events have likely involved a larger multiplicative factor, and longer and longer wavelengths. Could we see these spent photons? Any fraction of the CMB involved in dissipative interactions would likely have been shifted out of the black body curve. How far out in the spectrum do such effects extend? Our spectroscopies allow us to measure a substantial range of the electromagnetic spectrum involving interaction with electrons in bonds (in UV/VIS spectroscopies, bond vibrations (in the IR and Raman), or with spin flipping of electrons (in the microwave range, \( \sim 10 \) GHz in X-band EPR) or nuclei. For nuclear spins (NMR, ENDOR, ESEEM, etc.) interaction energies are in the 10-500 MHz range. Radio frequency measurements extend out as far as ELF photons with wavelengths in the 10-20 km range. Charge displacements associated with dielectric response extend from the refractive range out to seconds\(^63\); these are of interest in facilitating electron transfer reactions in molecular mechanisms, but one might imagine that they could also provide antennae for pho-tonic displacements. Beyond that we are, as it were, in the dark. Even when processes are not dissipative, as in refractive events, an additional consideration comes into play. Although \( E \) is returned in full in each successive event, momentum is borrowed from time for the duration of each interaction, and transferred to the refractive medium, summing to measurable transfer as the path-length increased, put to good effect in molecular tweezers, light sails, etc. Does momentum transfer have to involve charged species? Do we know of any data pertaining to unexplained pushing effects on massy objects?

In extending this line of thinking, the dark energy, which accounts for \( \sim 70\% \) of the mass/energy of the present cosmos and provides a pushing force acting to expand the universe, looks like an obvious candidate. As noted above, an unknown fraction of the mass/energy of the early cosmos has been converted to photon energy. Given the mechanism discussed above, it seems quite reasonable to expect this to be in the same ballpark as the fraction accounted for by dark energy. A fraction of the contribution to the push could be due to momentum applied to displacement of mass by spent photons, but how large that fraction might be depends on how open the universe is.
Is gravity quantized? Gravitons are listed in tables as massless neutral fundamental particles with spin 2, travelling at the speed of light. Since no one knows what gravitons are, we might stretch our minds further to a notion seen by experts as a fringe oddity, - the idea that the spin 2 of gravitons might be wrongly assigned, and that they are particles with spin of 1 like photons. This case has been made by Nigel Cook (cf.\textsuperscript{64}) and argued vigorously along the lines below; his engagement in several running battles with experts suggest he has little regard for authority. Considering the paucity of data from which properties can be determined, at least in one respect, the equivalence between photons and gravitons has been strongly supported by recent measurements in which LIGO observations of gravitational waves from collapse of binary neutron stars (GW 170817) have been correlated with photon arrival. After identification of the astronomical object involved, these have included measurements through a broad range of the electromagnetic spectrum of kinetics of phenomena associated with evolution of the host galaxy after the collapse, starting \(\sim 10\) hours after the LIGO signal. Most importantly for the present argument, a gamma wave burst (GRB 70817A) had been detected, and its time established independently, and found to 1.74 s after the LIGO observed merger time, and thus correlated with the gravitational wave by this coincidence. Given the distance of travel, the correlation allowed a very precise determination of relative velocities, giving \( -3 \times 10^{-15} < (v_{GW} - v_{EM}) < +7 \times 10^{-16} \) relative to the speed of light. As far as I understand it, this is the only empirical evidence establishing precisely a property for gravitons, and therefore should carry some weight. If gravitons are photons, and the spent photons contribute to the dark energy pushing the universe apart, then gravity would have to be a pushing force. We would then have to see the role of the massy universe (dark matter and “visible” mass) as providing a shielding effect; Cook\textsuperscript{64} provides a detailed quantification of the forces involved, which explains why gravity appears to act like a pulling force. Amazingly, if gravity is quantized, and gravitons are spin 1 particles, this scenario would also provide a clear and direct relation of gravity to quantum mechanics. Could the experts be wrong?

Conversion of the mass/energy of the universe to photons increases entropy through dissipation, dilution, temperature change and increase in the number of photon states. From their association with evolution of the cosmos, these entropic effects must be significant factors determining Eddington’s arrow, and therefore bring an understanding of the direction of time in terms of physical process leading ultimately to the entropic death of the universe. However, it might be
worth noting that there would be (in the current model) a countervailing ordering process. Since black holes are collecting light that can’t escape (except by Hawking’s evaporation), they could serve to recycle the energy of those spent photons as new shiny hot ones. This could happen if the supermassive black holes (SMBHs) at galactic centers were to explode. If the universe is constrained in volume (which might be the case if the spent photons haven’t diffused away), maybe over a long time, the remaining black holes could come together into a super big one. As more and more spent photons are sucked up, their pushing effect would be weakened, and the restraint on expansion might weakened, eventually allowing it to explode.

The naïve view that dark energy comes largely from spent photons needs to be tempered by another consideration. The discussions of the CMB radiation and of photons in the earliest phases of the big bang expansion are both framed in the context of their thermal equilibrium with the prevailing temperature. Clearly, gravitons do not fit into this picture; if they are photons, they are not thermally equilibrated. However, if gravity is quantized, in the steady-state, exchange between gravitons and mass displacement is a two-way affair; momentum changes associated with cosmic motion must lead to an outward flux of gravitons from moving bodies. As Cook points out, the isotropic push from the cosmos is likely to far outweigh the local flux outwards on the stellar scale. However, events like blackhole formation involve very large-scale mass displacements occurring rapidly, and might be expected to be detectable. Indeed, the studies on GW 170817 might be seen as demonstrating just such behavior. The LIGO data show a kinetics for the event lasting ~100 s, with oscillations starting with a frequency of 24 hertz, increasing in amplitude and frequency to a few hundred hertz, and ending in the typical chirp pattern on collision. This pattern shows the last moments of the spiraling-in of the two neutron stars, and their collapse to singularity, the latter (judging by the lack of any neutrino emission) occurring in milliseconds. Both phases involve enormous mass displacements. The LIGO apparatus detected this pattern of events through interference effects associated with the differential displacements of separate masses in the two orthogonal branches of an interferometer. As the pulse of gravitons arrived here it gave the masses a differential push. Obviously, much additional information can be gained from such data. The massive flux of visible photons from the accretion at the event horizon of black holes allows calculation of rotational rates and mass-conversion energies, and other cosmic clocks (pulsars for example) allow similar calculations, so perhaps the invisible
flux of gravitons might also be calculable. If one adopts the quantized view, the pulse of gravitons would radiate out uniformly from the initiating event, with an attenuation proportional to the ratio of surface area between that event, and the current wave-front. Since distance of the event can be estimated, this would allow trivial calculation of the attenuation in intensity. If the LIGO event allows calculation of the force applied in moving the masses, it would then tell us the force at the source, allowing comparison with Newtonian calculations from events at the source. One waits in anticipation that the LISA satellite system will provide data on many additional sources. Meanwhile, the above discussion raises some intriguing questions. What range of frequency spectrum do gravitons occupy? How can we account for the fact that gravity has the same accelerating effect in masses from small to large? If gravitons are not photons, what are they?

Switching to a different topic, the recent characterization of quasars from SMBHs dating back to the early evolution (red shifts in the range 6-7.54) has opened another tantalizing possibility. The oldest of these, quasar J1342+0928, dating back to 690 My after the big bang, was showing its activity about half way through the reionization which allowed light to reappear as stars began to form after the dark ages. Analysis of the IR spectrum shows that its mother galaxy is more metal rich and dust heavy than any non-quasar galaxies at similar red shift, and similar signatures have been seen in other SMBHs. In general, mapping these SMBHs onto the microwave background shows an approximately even distribution; given their age, it is perhaps not surprising that they have distributed in synchrony with the CMB. Some of the other properties are not readily explained by the current model. The size of the black hole (800 million solar masses (M☉)) begs the question of how long it took to grow. Modeling this and similar SMBHs suggests that “…black hole ‘seeds’ more massive than 1,000 M☉ by z= 40 are necessary to grow the observed supermassive black holes”. This date for the seeds, z = 40, is long after CMB photons escaped (at z = 1100), but long before reionization allowed any substantial star formation (at z ~11). One possibility is that the “seeds” are primordial black holes from the pre-deionization phase, but most discussion of these revolves around much smaller masses. Alternatively, one might speculate that these “seed” galaxies were the remnants of a previous universe, which also housed the monster progenitor of the big bang. After the event, the evolving new universe engulfed these embers of the dying one to feed and rekindled them, generating the quasars by which we know of them now.
It is worth noting that another “ordering” principle, evolution of life, is at work in the universe, locally represent by our biosphere. Evolution involves work invested in order (negentropy), and leads to more complex forms, and eventually civilizations. An interesting feature comes from engagement of the biosphere in recycling processes, which leads to a delay in the entropic erosion, arising from the time it takes for photosynthetic product to percolate through the biosphere. The extent of the delay reflects the complexity of the biosphere. The energy invested in the entropic delay sustains the biosphere. Looking at our local condition, until a few centuries ago, in the steady-state, the solar energy intercepted by photosynthesis in the biosphere was practically all returned as heat, to join the inanimate dissipation in giving the equation relating the multiplication of photons to their thermal equilibrium with emitter states discussed above. Although the fraction of the total solar flux invested in the biosphere is small, these entropic delays have maintained a continually evolving biosphere in an ordered state out of thermal equilibrium, with an incremental increase in negentropy over time. This negentropic investment has brought us to our present state. Perhaps inevitably, this incremental progress is now being perturbed by anthropocentric interventions. Over the last two centuries, the steady-state balance has been disturbed as more and more energy from burning of fossil resources has been poured into the world, most of it dissipated as heat. However, some of the energy has been diverted to construction; it is invested in the structure of our civilization, representing an additional negentropic contribution. Because of the enormous amplification of power arising from our mastery of technology, man is now responsible for energy fluxes (total world energy consumption by humankind) around a tenth of the flux through the biosphere. The downside to this growth in civilization has been an enormous increase in population, mass extinction of other species, a wasting of our environment, and most immediately worrying, climate change due to excessive generation of green-house gases. Stepping back from this local rapine, and extrapolating to a wider perspective, other life forms around us might well weather, or have weathered, the sort of crassness that threatens our own planet, and through advance in technology been able to exploit a larger local volume of space for material resources. Speculation on how significant such forces might be on cosmic timescales at present provides the background theme in many science fiction movies. Such exploration necessarily involved exploration further and further from the planetary home. If the speed of light is the limit, exploration outside the locale of a star system would be limited by the life expectancy of the species; if outside a galaxy, exploration by living beings is
perhaps an unlikely scenario given the times involved. More likely would be exploration by engineered self-replicating intelligences, or perhaps a symbiosis between organic and engineered life forms. In either case, to be self-sustaining, the vehicles would have to be quite massive, but equipped with robotic agencies capable of local action, for example in mining operations, etc. They would then be able to carry out into the cosmos the accumulated wisdom of the civilization(s) that engendered them. Over the eons, who knows if these intelligences might not get together and use their advanced technologies to steer the last stages of an aging universe to a future more interesting than entropic death.

“…Azrael, who knew the secrets, thought: ‘I remember when all this will be again’…”; with this enigmatic phrase, Terry Pratchett closes Reaper Man68 (book 11 in the Discworld series). I borrow it here in homage, and to make a fitting ending.

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Fig. S1_1. Analysis of the separate PDC processes in a polarization Sagnac interferometer.  

A. Scheme of interferometer. The pump laser, after rotation to 45° is directed to a polarizing beam splitter (PBS) by a dichroic mirror (reflecting the pump (405 nm) but not the PDC output (810 nm). The PBS splits the beam into $H$ (orange, $E_{H}$ in 40°) and $V$ (cyan, or $E_{V}$ in 40°) components, shown by the solid lines. These enter the interferometer and pass through it in counter-clockwise and clockwise paths, respectively. The $E_{V}$ beam is intercepted by HWP2, which rotates it by 90°, from $V$ to $H$. Two mirrors reflect the beams so as to pass through the PDC crystal (periodically-poled KTiOPO$_4$) from opposite directions, but now both $H$ oriented. The two output beams are co-linear, but are shown here as parallel dotted or dashed lines, initially $H$ (orange, signal) and $V$ (blue, idler), are directed back to the polarizing beam-splitter by the two mirrors. However, the $H_{S}$ and $V_{I}$ output in the counter-clockwise path (dashed lines) passes back through HWP2 so that the signal beam becomes $V$ and the idler $H$ to give a $V_{S}$ and $H_{I}$ orientations. In the clockwise path, the output beams (dotted lines) retain their initial $H_{S}$ and $V_{I}$ orientations. At the PBS, the output beams are recombined so that $H_{S}$ (orange, dotted) and $V_{S}$ (cyan, dashed) beams go to station 1, and $H_{I}$ (orange, dashed) and $V_{I}$ (cyan, dotted) beams go to station 2. Note that all output beams are fully polarized, and that at each station the two beams come from different PDC processes.  

B. Simulation of the clockwise process. Default settings are used, but the PDC_EO photon mode was selected so that the Ordinary (signal, $H_{S}$) output beam was directed to station 1, and the extraordinary ($V_{I}$, idler) beam to station 2. In the left panel, the outcome from an anti-correlation count shows a standard LR0 curve. In the right panel, the singles counts at the four detectors shown the polarization of the beams. At station 1, with the polarizer fixed at 0°, all the photons (in the $H_{S}$ beam) pass to the ordinary ray ($Q$ detector, red symbols), and there are no photons detected in the extraordinary ray ($R$ detector, blue symbols). At station 2, with the variable polarizer, the distribution of photons (in the $V_{I}$ beam) followed complementary sinusoidal curves at the $S$ (yellow symbols) and $T$ (green symbols) detectors as the angle difference was varied, as ex-
pected from Malus’ law. At both stations, the mean of the singles counts was 0.5, because, at ei-
ther station, the sum always reflected a complementary distribution of $H$ and $V$ at the two detec-
tors, and the overall mean was 0.5 (black symbols).

C. Simulation of the counter-clockwise process. The set up was the same as in B, but HWP0
(simulating HWP2 in A) was inserted to rotate both output beams by 90° so that the signal beam
(ordinary ray) photons contained all $V$ photons, $V_S$, and was directed to station 1, and all $H$
photons ($H_I$ in the idler ray) to station 2. In the left panel, the same LR0 outcome is seen, but in the
right panel, the singles counts showed the opposite distribution to that in B. At station 1, all the $V$
photons are passed to the extraordinary ray ($R$ detector, blue symbols), and no photons are de-
tected at the $Q$ detector (red symbols); at station 2, with the variable polarizer, the complemen-
tary sinusoids at the $S$ and $T$ detectors showed the opposite distribution. As in B, the mean of the
singles counts at each station was 0.5, as was the overall mean.

D. Simulation of the Sagnac interferometer. The outputs from successive runs, one as in B, the
second as in C, are displayed together. The two LR0 curves overlap (they would sum in the out-
come of the Sagnac interferometer). Station 1 now receives an equal mix of $H_S$ and $V_S$, and sta-
tion 2 an equal mix of $V_I$ and $H_I$ photons, so that, although the orthogonal populations came from
separate PDC processes, the incoming flux at both stations would appear to be dichotomic. The
consequence of this behavior in real experiments would be that the polarization effects shown in
B and C would cancel, so that the polarized state would be undetectable. However, the simula-
tion shows that the “interference” is not due to quantum “magic”, but to simple cancellations.

Fig. SI_2. Simulation of experiments showing full-visibility outcome on changing the orien-
tation of the fixed polarization analyzer.

A. Simulation of outcome independence on the rotational frame in polarization Sagnac interfer-
ometer configuration (cf.18,40,42). In simulations similar to those in Fig. SI_1D, the orientation of
the reference “polarization analyzer” was changed from 0° to 45° or -45°. This was achieved by
setting HWP1 to rotate the beam by 0° (or 45°, or -45°), and polarizer 1 to 0° (or 45°, or -45°). As
expected from Malus’ law, the effect was to shift the phase of the outcome curve by the angle
chosen, while retaining full-visibility (left panel; closed circles are without HWP0, open triangles
are with HWP0 set to 45° to rotate both beams by 90°). Setting of HWP1 alone to rotate the
beam by 45° or -45°, or polarizer 1 alone to 45° or -45°, resulted in a zero visibility outcome
curve (see main text, Fig. 8). As long as the polarizer was set to match the beam rotation, the same full-visibility outcome curve was generated, phase-shifted by the angle chosen. The right panel shows, as expected, that the same phase shifts were seen in the sinusoidal curves for singles counts at station 2. The phase shifts are due simply to the shift in phase of the reference frame.

**B. Simulation of outcome independence on the rotational frame with conventional PDC configuration (cf. 38, 39).** The same manipulations of “polarization analyzer” settings as in A were used to simulate the experimental results claimed to eliminate local realistic models. The phase-shifted full-visibility curves in the left panel, generated under vOL constraints, show that the experimental results can be explained by a local realistic model. The singles counts (right panel) show no polarization effects because in these configurations the beams at each station contained an equal mix of $H$ and $V$ photons from the cone intercepts.

**Fig. SI 3.** Photographically recorded diffraction pattern of a pinhole aperture of 90 $\mu$m, illuminated with red laser light (~650 nm) at a distance of 65 millimeters, in which the zeroth (diffraction plates in the middle) to the 27th diffraction order (upper left) can be seen (from Wikipedia – Airy Disk).
Fig. SI_1A
Fig. S1_1D
Fig. SI-2A
Fig. SI-2B
Figure SI_3